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BULLETIN NO. 20

TESTS OF CONCRETE AND REINFORCED CONCRETE COLUMNS

SERIES OF 1907

BY

ARTHUR N. TALBOT

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 20 DECEMBER 1907

TESTS OF CONCRETE AND REINFORCED CONCRETE COLUMNS; SERIES OF 1907

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS

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I. Introduction

1. *Preliminary.*—In structures of reinforced concrete, columns and other members subject to direct compressive stresses are used. Usually these members are reinforced with longitudinal rods, and quite generally light ties connect these rods at intervals, though tests indicate that with the size and spacing usually adopted these ties add little or no strength to the column. Much interest has been manifested recently in columns reinforced with steel hooping in such a way as to give lateral restraint to the concrete. A variety of opinions has been expressed on the action and on the value of this type of reinforcement. Tests which have been reported show that the hooped column has great strength, though the results of the tests have not been conclusive on its applicability to general construction. In Bulletin No. 10 of the University of Illinois Engineering Experiment Station, Tests of Concrete and Reinforced Concrete Columns, Series of 1906, there were given the results of tests made on plain concrete columns and columns with longitudinal reinforcement. Further tests of plain concrete columns have now been made, and also tests of hooped columns. This bulletin will give the results of these tests, and it is hoped that it will throw some light on the properties of both plain and hooped concrete. It may be added that further tests are now in progress.

2. *Acknowledgment.*—The investigations herein described were made in the Laboratory of Applied Mechanics of the University of Illinois as a part of the work of the University of Illinois Engineering Experiment Station. Much of the work of making the tests on the columns was done as thesis work by Sidney Grear, Charles E. Hoff, and Clyde L. Mowder, senior students in Civil Engineering, class of 1907. Credit is due them for the thought and care given to this work. The immediate supervision of the work of making the columns and of the work of conducting the tests was given by D. A. Abrams, Associate in the Engineering Experiment Station. Many of the tests included in this work were made by Mr. Abrams and Mr. W. R. Robinson, First Assistant in the Engineering Experiment Station, and their aid in this and in interpreting the results and assisting in the preparation of this bulletin has added much to the value of the work. The bands used for hooping were furnished by Mr. Robert A. Cummings, Pittsburg, Pennsylvania, and were made by

The Electric Welding Company of Pittsburg, Pennsylvania. The wire spirals used for reinforcement were furnished for these tests by The American System of Reinforcing for Concrete Construction, of Chicago.

3. *Scope of Bulletin.*—Nineteen plain columns and thirty hooped columns were tested. The columns were generally cylinders 12 in. in diameter and 10 ft. long, but a few of other dimensions were used. The ratio of length to diameter ranged from 6 to 13, all being short columns. Tests were usually made at an age of 60 days, but a few tests were made at 6 months. In addition to the above, two columns remaining from the series of 1906 were tested at ages of 12 and 16 months. In the plain concrete columns tested, the strength of the column, the relation between the load and the deformation and the resulting modulus of elasticity have been determined. An effort was made to measure Poisson's ratio for concrete. For the hooped columns, both bands and wire spirals were used for reinforcement. Columns reinforced with bands will be referred to as band-hooped columns and those reinforced with wire spiral as spiral-hooped columns. The bands were of three weights: No. 8, No. 12, and No. 16 gauge, all 1 in. in width. The wire used in the spirals was No. 7 and $\frac{1}{4}$ in.; both high-carbon and mild-steel wire were used. The concrete was generally a 1–2–4 mixture, but in order to determine the effect of the restraint of the hoops upon richer and leaner concrete, two columns were made with 1–4–8 concrete and two with 1–1½–3 concrete. A feature of the tests of the hooped columns is the exhibition of shortening and of toughness. In the tests of the hooped columns, measurements were made to determine the amount of shortening due to the load. An attempt was made to determine the amount of the stress in the steel hooping. To throw a little light on the general phenomena, the concrete from one of the columns was tested after it had been loaded and then stripped of the spiral, and one of the columns was loaded eccentrically.

4. *Notes on Hooped Concrete.*—While it has long been known that lateral restraint adds to the resistance of materials in compression, M. Armand Considère, Inspecteur Gènèral des Ponts et Chaussées of France, was the first to utilize, develop, and investigate the construction formed by restraining the concrete by means of bands and spirals now generally known as hooped con-

crete (béton fretté). Experiments were made by Considère in 1902 which were communicated to l'Academie des Sciences in August and September 1902. A more extended account of these experiments was published in Genie Civil.* The tests of 1902 and 1903 were made upon octagonal columns, 5.9 in. short diameter, reinforced with spirally wound wire and longitudinal rods. In November 1904 a bridge of 65.6 ft. span, built for experimental purposes, was tested at Ivry, Paris. The tests and analyses of hooped concrete were given by Considère in a work describing his researches in reinforced concrete.†

In 1907 Considère published a monograph, "Le Béton Fretté et ses Applications", in which were collected and discussed various experiments on hooped concrete. In this Considère gave his revised conclusions on the properties and applicability of this form of reinforced concrete. A principal table gives the results of a part of the experiments made by Bach at Stuttgart and the second and third sets of experiments made by Considère under the direction of the Commission du Ciment Armè. These columns were octagonal and square, 200 and 275 mm. in diameter, and 1 m. and 4 m. long. The amount of the added strength given by the hooping and longitudinal reinforcement, as shown in this table, ranges from 525 to 880 lb. per sq. in. for each 1% of total reinforcement, based on the gross section of the column. The average value was 710 lb. per sq. in. Based on the area of the hooped core this would be about 880 lb. per sq. in. Another test cited is that of a column 700 mm. in diameter and 1.4 m. in length, reinforced with hooping equivalent to 2.73% of the concrete core, and having longitudinal reinforcement equal to 1.44% of the cross section. This column, which was tested by Professor Guidi, of Turin, carried 1 620 000 kilograms, or 6000 lb. per sq. in. of the gross area of the column, and 7650 lb. per sq. in. of the hooped core. Tests made with columns reinforced with three concentric helices gave very high results.

Considère considers that between the molecules of hooped bodies there is produced a phenomenon analogous to the frictional resistance in sand boxes employed for carrying the centers of bridges during construction, but he adds that cohesion in hooped

*Genie Civil, v. XLII. No. 1, 2, 3, 4, 5, 6, and 9.

†Experimental Researches on Reinforced Concrete, Considere, translated by Leon S. Moisseiff, McGraw Publishing Company, New York.

bodies persists under pressure in spite of large deformations. In order to produce the maximum resistance the hooping should fill the following conditions: The hoops should be near enough together and should form a net-work so close that the concrete cannot escape through the meshes. In the earlier writings, he gives the maximum allowable distance between the spirals as $\frac{1}{5}$ to $\frac{1}{10}$ of the diameter of the helix. The hooped core should be full or not present voids of importance. The projection of the hoops on a plane perpendicular to the axis of the prism should be convex. He modifies the conclusions given in 1902 by attributing the increased strength to an increased cohesion of the particles as well as to intermolecular friction.

Considère gives the following rule for the resisting strength of hooped columns: The compressive resistance of hooping members exceeds the sum of the following three elements, (1) the compressive resistance of the concrete core without reinforcement increased by 50% of itself, (2) the resistance of the longitudinal reinforcement stressed to its elastic limit, (3) the compressive resistance which would be produced by longitudinal reinforcement at the elastic limit of the hooping metal and equal to 2.1 times as great a volume as that of the hooping metal. He considers the factor 2.1 as very low but adopts it in accordance with the instructions issued by the Minister of Public Works of France, October, 1906.* He finally gives the following formula for computing the strength of a short hooped column,

$$C = 1.5\ c + 2400\ p + 5100\ p'$$

where C is the unit strength of the hooped column, c is the unit strength of a plain concrete column, p is the ratio of the longitudinal reinforcement to the concrete core, and p' is the ratio of the hooping to the concrete core. The area of the column is taken as that of the concrete core. All loads are here given in pounds per square inch. The formula above gives results which agree closely with the experimental values cited in the monograph.

Considère considers that hooped concrete has many advantages for numerous constructions. The case of posts and columns is given as an important application and it is considered that hooping will permit the use of leaner concrete or will allow a re-

*Genie Civil, v. 50, p. 177.

duction of cross section. In beams the hooping may be utilized to increase the resistance to compression. Piles, compression pieces in bridges, and other forms of struts will show points of advantage. A strong feature is stated to be its extreme ductility and toughness and the fact that it exhibits signs of weakness before failure. The extreme shortening found is not considered troublesome, especially where the structure is articulated. He recommends a rather low factor of safety and considers that the experiments made have established the utility and safety of hooped concrete as a form of building construction. As before noted, the Minister of Public Works of France issued instructions authorizing the use of this form of construction.

Important tests on hooped concrete were made by Professor C. Bach, Director of the Materialprüfungsanstalt der Kgl. Technischen Hochschule in Stuttgart in 1905. The results of these tests were published in Mitteilungen über Forschungsarbeiten, No. 29, and are discussed in Morsch's Eisenbetonbau, page 69. The columns externally were octagonal in form with a short diameter of 275 mm. and a height of 1 m. The circles of the spirals had a diameter of about 235 mm. Longitudinal reinforcement was also used. Two sets of tests were made in which the pitch of the spiral was more than one third of the diameter of the core, and the increase of strength of these columns over that of plain concrete was only about half as much per 1% of reinforcement as was found in columns having the pitch of the spiral less than $\frac{1}{6}$ of the diameter of the core. The tests of the columns reinforced with the small pitch gave an increase in strength over plain concrete amounting on an average to 485 lb. per sq. in. for each 1% of total reinforcement used, when based upon the gross area of the column and 755 lb. per sq. in. when only the area within the hooping is considered. The range in the latter case is from 412 to 1070 lb. per sq. in. The results do not show very definitely how much of the increased strength is due to longitudinal rods and how much to spirals. In the monograph referred to above, Considère considers that only one set (9 columns out of 57) followed the requirements given in the instructions of the Minister of Public Works of France, and he omits the others from his analysis. The remaining columns, however, give considerable information.

Tests of Metals, 1906, gives the results of tests of hooped

columns at the Watertown Arsenal by James E. Howard. These tests include quite a variety of reinforcement, with bands, spirals, and longitudinal bars and angles, and seem to indicate that the strength added by 1% of spiral reinforcement ranges from 600 to 1000 lb. per sq. in., based on the area within the hooping.

II. Materials, Test Pieces, and Methods of Testing.

5. *Materials.*—The materials used in making the columns were similar to the best materials used for this class of work in this section of the country. The stone, sand, and nearly all the cement were bought in the open market. Universal portland cement, used in two columns, was furnished by the makers, but the Chicago AA portland cement used in the remainder was bought of a local dealer. The bands were supplied by Robert A. Cummings of Pittsburg, Pennsylvania, and were made by The Electric Welding Company of Pittsburg, Pennsylvania. The American System of Reinforcing for Concrete Construction, of Chicago, furnished the steel spiral reinforcement.

Stone.—The stone was crushed limestone from Kankakee, Illinois. It was ordered to pass through a 1-in. screen and over a $\frac{1}{4}$-in. screen. The stone came in two shipments, and there was an appreciable difference in the quality of the two. The first lot was finer, softer, and of a darker color than the second lot. The harder, coarser stone of the second lot made much the better concrete. Tests showed that this last lot had about 50% voids and weighed 85 lb. per cu. ft. loose. Table 1 gives the average of several mechanical analyses of stone from the second lot.

TABLE 1.

Mechanical Analysis of Stone.

Size of Mesh	Per cent Passing
$1\frac{1}{4}$-in.	100
1 "	95
$\frac{1}{2}$ "	35
$\frac{1}{4}$ "	5
$\frac{1}{8}$ "	2

Sand.—The sand was of good quality, sharp, well graded, fairly clean, contained 28% voids, as determined by pouring sand

into a vessel partly filled with water, and weighed 103 lb. per cu. ft. loose. It came from near the Wabash River at Attica, Indiana. The result of two mechanical analyses of this sand is given in Table 2.

Cement.—In all except two columns, Chicago AA portland cement was used. In two columns Universal portland cement

TABLE 2.

MECHANICAL ANALYSIS OF SAND.

Sieve No.	Separation Size inches	Per cent Passing	
		First Trial	Second Trial
5	.174	99.6	99.6
10	.091	83.1	79.8
12	.067	74.8	71.0
16	....	67.4	62.5
18	.043	56.0	50.6
30	.027	36.7	32.9
40	.019	25.5	22.4
50	.013	14.5	13.7
74	.009	7.2	6.5
150	....	1.8	2.0
200	....	1.3	1.0

TABLE 3.

TENSILE STRENGTH OF CEMENT.

Ref. No.	Ultimate Strength, lb. per sq. in.							
	Chicago AA Cement				Universal Cement			
	Age 7 days		Age 28 days		Age 7 days		Age 28 days	
	Neat	1-3	Neat	1-3	Neat	1-3	Neat	1-3
1	786	230	811	265	410	187	680	370
2	683	145	851	235	470	200	670	330
3	760	218	861	225	360	120	560	360
4	786	186	760	270	405	145	570	290
5	866	215	965	265	320	195	600	295
6	815	211	935	287	310	180	620	310
Av.	783	201	864	258	379	171	617	326

was used. Table 3 gives the results of tension tests of these cements made according to standard practice.

Concrete.—Men accustomed to mixing concrete made the test specimens and an effort was made to have the concrete as good as would be used in the best practice. All materials were pro-

TABLE 4.

TENSION TESTS OF STEEL USED IN COLUMNS.

The values given for "Weld not in tested section" are averages of several tests.

Nominal Size or Number	Area of Section sq. in.	Yield Point pounds	Maximum Load pounds	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Per cent Elongation in 8 in.	Remarks
¼-in.	.049	5650	7430	115 500	152 000	1.5	High-carbon spiral
7	.024	1430	2170	59 700	90 500	12.2	High-carbon spiral
¼ in.	.046	2500	3380	54 100	73 000	15.5	Mild-steel spiral
7	.025	940	1310	38 000	53 000	16.4	" " "
			Weld not in tested section.				
16	.063	3160	4270	50 300	68 100	21.5	9-in. bands
12	.127	5920	8160	46 600	64 300	17.5	" "
16	.064	3000	3830	46 700	59 700	18.0	12-in. bands
12	.127	5460	7710	46 900	60 800	17.5	" "
8	.180	8930	11640	49 800	65 000	20.2	" "
			Weld in tested section.				
16	.064	3010	3040	46 700	47 100	1	9-in. band. Broke at weld.
16	.062	3150	4190	50 600	62 400	15	9-in. band. Did not break at weld.
12	.137	6250	8300	45 900	60 900	7	9-in. band. Broke at weld.
12	.123	6160	8150	50 100	66 300	14	9-in. band. Did not break at weld.
16	.065	.....	2730	.....	41 900	.....	12-in. band. Broke at weld.
12	.121	5700	7080	47 300	58 700	7	12-in. band. Broke at weld.
8	.188	9300	12120	49 300	64 400	17	12-in. band. Did not break at weld.

portioned by loose volume and weights were taken as a check on the measurement. The mixing was done with shovels by hand. The sand and cement were first mixed dry. The stone, which had previously been thoroughly moistened, was then added and the mass turned until uniform in appearance. Water was added in such proportion as to give a fairly wet mixture. This consistency permitted tamping into the forms to good advantage.

Steel.—The steel used in the hoops had a yield point of about 48 000 lb. per sq. in. and an ultimate strength of from 60 000 to 65 000 lb. per sq. in. The hoops were 1 in. wide and of three thicknesses, corresponding to No. 8, 12, and 16 gauge. For the spiral reinforcement, both high-carbon and mild-steel wire were used, each being in two sizes, $\frac{1}{4}$-in. and No. 7. Tests showed that the $\frac{1}{4}$-in. high-carbon wire had a yield point of approximately 115 000 lb. per sq. in. The yield point of the No. 7 high-carbon wire was about 60 000 lb. per sq. in. Their ultimate strengths were about 155 000 and 90 000 lb. per sq. in., respectively. The $\frac{1}{4}$-in. mild-steel wire had a yield point of 54 000 lb. per sq. in. and the No. 7 wire 38 500 lb. per sq. in. Their ultimate strengths were about 73 000 and 54 000 lb. per sq. in. respectively. Table 4 gives the results of tension tests made on the reinforcement, some of the specimens being taken from tested columns and some from unused material. The hoops were electrically welded and as it was desired to know the strength of the weld, pieces containing the weld were tested. Several broke at the weld at a point much below the ultimate strength, but with one exception at a load higher than the yield point of the metal.

6. *Test Specimens.*—An effort was made to have the conditions of fabrication of the different test specimens as nearly uniform as possible. Usually, the test columns were made in sets of three. Individual columns of a given set were made at different dates throughout the season, the purpose being to make columns of different sets at the same time and thus distribute accidental variations of fabrication over a number of sets. To give a check test on the quality of the concrete, 12-in. cubes, 6-in. cubes, and an 8 x 16-in. cylinder were made in many cases from the batch for a column. The 12-in. cubes were made in sets of two, and the 6-in. cubes in sets of three. The mixture varied from 1–1$\frac{1}{2}$–3 to 1–4–8, by far the largest number being of a 1–2–4

mixture. As noted above, there was a difference in the stone which affected the strength of the test specimens, those made of coarser and harder stone being the stronger. General data on all test specimens are given in Table 5, page 14.

Plain Concrete Columns.—Nineteen columns containing no reinforcement were tested. These columns were cylindrical, 12 in. in diameter and 10 ft. long. In two columns Universal portland cement was used; in the others Chicago AA was used. Two columns not included in the nineteen named above, No. 21 and 22, were from the lot made in 1906 at the same time as those described in Bulletin No. 10, and the materials, method of making,

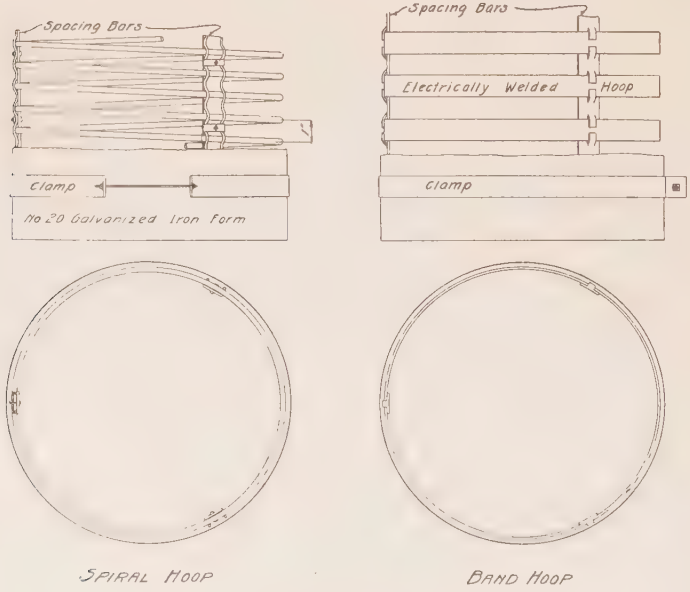

Fig. 1. Forms and Reinforcement.

and other conditions were given in that bulletin. These columns were 12 in. square and 6 ft. long, and were 12 and 16 months old when tested.

Band-hooped Reinforced Columns.—Twelve band-hooped columns were 12 in. in diameter and six were 9 in. in diameter. The length was 10 ft. The reinforcing consisted of circular bands of No. 8, 12, and 16 band steel 1 in. wide generally placed on 2 in.

centers. The spacing for one column was 3 in. centers and for two columns 4 in. centers. The bands were held in place by three longitudinal strips of thin metal suitably punched, as shown in Fig. 1.

Spiral-hooped Reinforced Columns.—Twelve spiral-hooped columns were tested. They were 12 in. in diameter and 10 ft. long. The reinforcement consisted of a single steel wire coiled in the form of a helix and spaced at a uniform pitch of 1 in. by means of heavy steel wires crimped as shown in Fig. 1. Both mild-steel and high-carbon-steel wire were used, the sizes being No. 7 and $\frac{1}{4}$-in. for each.

Cubes.—The cubes were made in two sizes, 12-in. and 6-in. These cubes were intended to aid in judging the quality of the concrete used in correspondingly numbered columns. The concrete for the cubes was taken from the middle of the batch for the correspondingly numbered columns and was thought to be representative. The concrete was well tamped in the forms and spaded around the edges to insure a good surface.

Cylinders.—The cylinders were all 8 in. in diameter and 16 in. high. The concrete was selected and tamped in the same way as for the cubes.

7. *Forms.*—The column forms were of galvanized sheet steel bent into a cylindrical shape and held in position by bands 1 in. wide and $\frac{3}{16}$ in. thick. The bands could be adjusted to the proper diameter by means of bolts. The forms were built in sections $2\frac{1}{2}$ ft. long and fitted together in stove-pipe fashion. The forms are shown with dimensions in Fig. 1. The forms for the 12-in. cubes were of the ordinary wooden type, two in a set; those for the 6-in. cubes were of steel, three in a set. The cylinder forms were of wrought iron with cast-iron bases, being the same as shown in Fig. 1 of Bulletin No. 10. All forms remained in place about ten days.

8. *Making of Columns.*—The concrete for each column and for the corresponding cubes and cylinders was mixed in one batch. The form was set up on a cast-iron base plate, 14 in. x 14 in. x $1\frac{1}{4}$ in., which was planed on both sides and served as a bearing plate in the column test. In making the plain concrete columns the forms were built up in $2\frac{1}{2}$-ft. sections, each section being filled before the next was added. The concrete was put in in layers of about 6

TABLE 5.

General Data of Test Specimens.

The length varied from 10 ft. 0 in. to 10 ft. 4 in. except for Columns No. 21 and 22, which were 6 ft. long.

Column No.	Diam. in.	Kind of Concrete	Kind of Stone	Reinforcement		Minor Specimens		
				Per cent	Size and Spacing	Cubes		Cyl.
						12-in.	6-in.	8-in. x 16-in.
101	12	1-2-4	Soft		None	2	...	....
102	"	"	Hard		"	...	...	....
103	"	"	"		"	2	...	....
104	"	"	"		"	2	...	....
105	"	"	"		"	...	6	1
108	"	"	"		"	2	3	1
109	"	"	"		"	2	...	....
110	"	"	"		"	2	...	....
111	"	1-1½-3	"		"	2	3	1
112	"	"	"		"	2	3	1
116	"	1-3-6	"		"	2	3	1
117	"	"	"		"	...	3	1
121	"	1-4-8	"		"	2	3	1
122	"	"	"		"	2	3	1
128	"	1-2-4	"		"	2	...	....
129	"	"	"		"	2	...	....
163	"	"	"		"	2	...	....
164	"	"	"		"	2	. .	1
168	"	"	"		"	1	...	1
21	12-in. sq.	1-2-3¾	.....		"	...	...	....
22	"	"	.....		"	...	...	....
131	12	1-2-4	Hard	1.08	No. 16 bands (mild steel) 2 in. cc.	2	3	....
132	"	"	"	1.08	do.	2	...	. . .
133	"	"	"	1.07	do.	2	...	....
136	"	"	"	2.08	No. 12 bands (mild steel) 2 in. cc.	2	3	....
137	"	"	"	2.08	do.	2	...	....
138	"	"	"	2.10	do.	2	3	1
141	"	"	"	1.02	No. 12 bands (mild steel) 4 in. cc.	2	...	...
142	"	"	"	1.03	do.	2	3	1
143	"	"	"	1.39	No. 12 bands (mild steel) 3 in. cc.	2	...	....
146	"	"	"	3.20	No. 8 bands (mild steel) 2 in. cc.	2	3	...
147	"	"	"	3.20	do.	2	3	1
148	"	"	"	3.20	do.	...	...	1
151	9	"	"	1.47	No. 16 bands (mild steel) 2 in. cc.	...	...	....
152	9	1-4-8	"	1.35	do.	2	3	1
153	9	"	"	1.41	do.	2	...	....
156	9	1-2-4	"	2.76	No. 12 bands (mild steel) 2 in. cc.	...	...	....
157	9	1-1½-3	"	2.94	do.	2	3	1
158	9	"	"	2.78	do.	2	...	....

TABLE 5.—*Continued.*

Column No.	Diam. in.	Kind of Concrete	Kind of Stone	Reinforcement		Minor Specimens		
				Per cent	Size and Spacing	Cubes		Cyl.
						12-in.	6-in.	8-in. x 16-in.
171	12	1-2-4	Hard	0.85	No. 7 spiral (high steel)	2	...	....
172	"	"	"	0.85	do.	...	...	1
173	"	"	"	0.82	do.	...	...	....
176	"	"	"	0.84	No. 7 spiral (mild steel)	2	...	....
177	"	"	"	0.85	do.	2	...	...
178	"	"	"	0.84	do.	2	...	....
181	"	"	"	1.63	¼-in. spiral (high steel)	...	...	1
182	"	"	"	1.67	do.	2	...	.. .
183	"	"	"	1.68	do.	...	...	....
186	"	"	"	1.63	¼-in. spiral (mild steel)	2	...	1
187	"	"	"	1.71	do.	2	3	....
188	"	"	"	1.62	do.	...	...	.. .

in. and tamped or churned until water flushed to the top. In making the reinforced columns, the form was placed around the hoops or spirals for the full length of the column, after which it was set up vertically on the base plate. The concrete was put in from the top of the column and tamped or churned as described above. The reinforcement was usually covered by from $\frac{1}{16}$ in. to $\frac{1}{4}$ in. of mortar. As this outside coat spalled off early in the test and did not add appreciably to the strength of the column, the diameter of the column was assumed to be the same as the diameter of the band or spiral. Generally the column had a very good surface. In the band-hooped columns care was taken to place the bands so that welds were distributed over the circumference of the column. Three or four inches of the ends of the wire in the spiral-hooped columns were bent inward to form an anchorage. The metal in the reinforcement was gauged in several places and the average taken to compute the per cent reinforcement. The lengths of the columns were quite uniform, varying from 10 ft. 1 in. to 10 ft. 4 in.

9. *Storage of Test Specimens.*—The columns were built near the walls of the testing room of the Laboratory of Applied Mechanics and remained where made until time of the test. (See Fig. 2).

The forms were taken off 10 days after making and after that the columns were sprinkled with water twice daily until they were tested. The temperature of the room varied from 55° to 70° F. The 12-in. cubes were stored in the open air of the same room. The 6-in. cubes and cylinders were stored in damp sand.

10. *Summary of Test Pieces.*—Table 5 gives a list of all the test pieces. Specimens having corresponding numbers were made from the same batch of concrete.

11. *Testing Machines Used.*—The machine used in testing the columns and 12-in. cubes was the 600 000-lb. Riehle vertical screw machine of the Laboratory of Applied Mechanics. The slowest speed, 0.05 in. per min., was used, except on two columns where repeated loading was tried. Here a speed of 0.10 in. per min. was used. Fig. 3 shows a column in the machine in position for testing. The 6-in. cubes and the cylinders were tested in Olsen testing machines of 100 000 or 200 000 lb. capacity.

12. *Measuring Devices.*—The instruments used for measuring the shortening of the columns were the same as those used in the 1906 series of column tests and are very well shown in Fig. 3 and 4 of Bulletin No. 10. The yokes were usually placed to give a gauge length of 100 inches. The arrangement was such as to give practically two independent sets of readings. The measuring device consisted of a steel drum one-half inch in circumference, which rotated as the contact-rod moved. On the axis of the drum was a pointer which passed over a graduated dial. This instrument could be read directly to thousandths of an inch, and by a vernier to ten-thousandths of an inch.

The instruments used to measure the lateral expansion of the column were of two types. In one type, eight standards, each having four points of contact on the surface of the column, were distributed around the column and held together by elastic bands; a fine wire passed from standard to standard and was attached at either end to a dial extensometer from which readings were taken. Different devices were used to reduce the resistance to movement of the wire. This instrument gave the general expansion around the column. In the second type, contact was made at two points at the ends of a diameter of the column and the movement was magnified by transmission to the end of lever arms and there read on an extensometer dial. Fig. 3 gives a view of these instru-

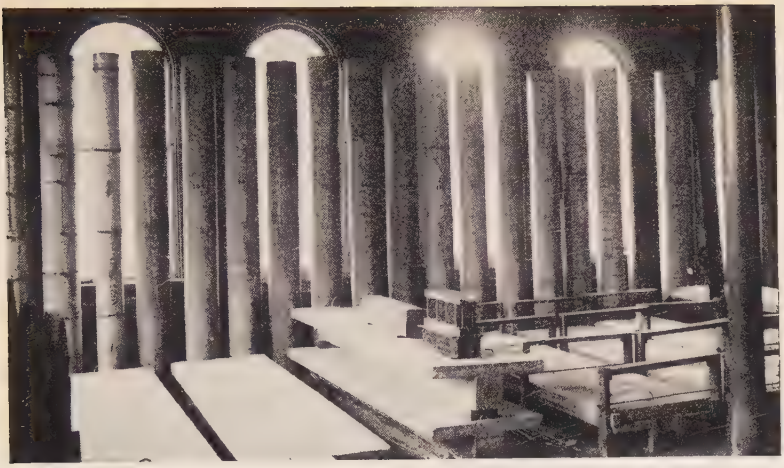

FIG. 2. VIEW SHOWING A NUMBER OF TEST COLUMNS.

FIG. 3. VIEW SHOWING COLUMN READY FOR TEST.

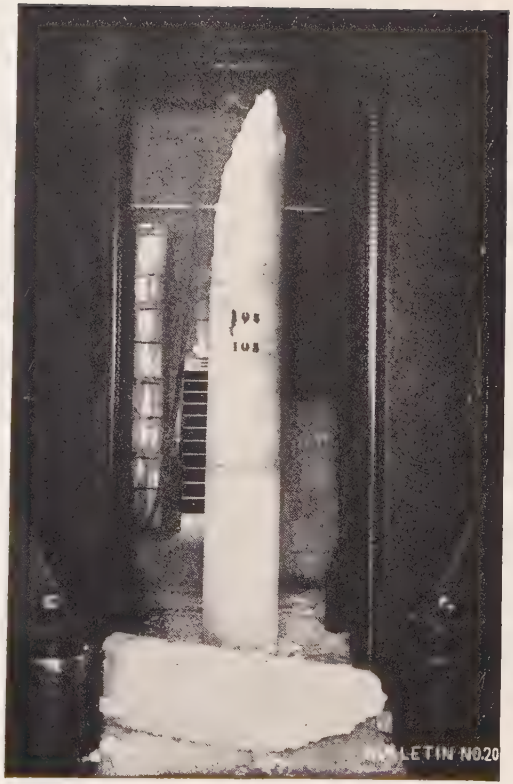

FIG. 4. VIEW SHOWING DIAGONAL SHEARING FAILURE OF PLAIN CONCRETE COLUMN.

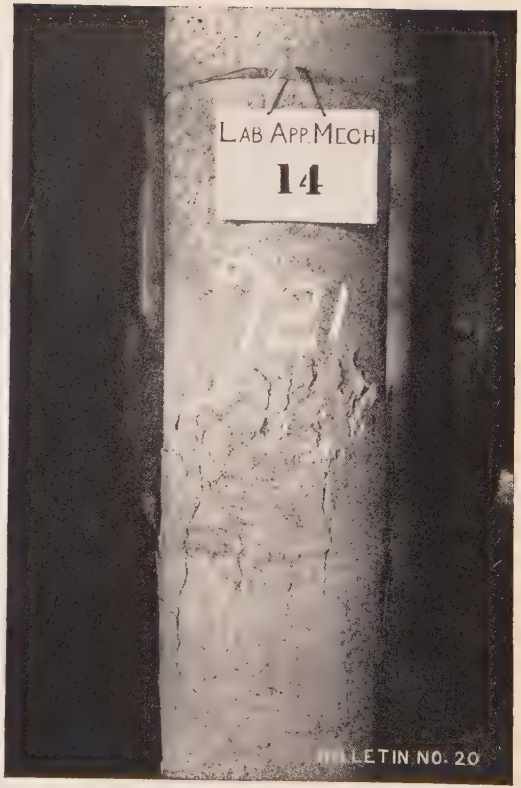

FIG. 5 VIEW SHOWING SIMPLE COMPRESSION FAILURE OF PLAIN CONCRETE COLUMN.

TABLE 6.

CUBE TESTS.

No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.	No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.
101_1	1–2–4	143.0	110	1890	122_1	1–4–8	144.0	79	1255
101_2	"	144.6	110	1875	122_2	"	146.2	79	1140
103_1	"	145.1	186	3120	122_3	"	36.0	288	1550
103_2	"	144.6	186	3560	122_4	"	36.0	289	1800
104_1	"	144.0	72	1960	122_5	"	36.0	303	1970
104_2	"	144.0	72	2110	128_1	1–2–4	144.0	200	1830
105_1	"	36.0	335	3750	128_2	"	144.8	200	1870
105_2	"	36.0	335	3200	129_1	"	142.5	200	1690
105_3	"	36.0	335	3840	129_2	"	139.8	200	1860
105_4	"	36.0	335	3320	163_1	"	144.8	185	2620
105_5	"	36.0	335	3280	163_2	"	144.0	185	2750
105_6	"	36.0	335	3970	164_1	"	145.1	184	2560
108_1	"	141.6	62	1880	164_2	"	144.6	184	2500
108_2	"	143.2	62	1850	168_1	"	144.8	185	2370
108_3	"	36.0	294	3180	131_1	"	144.0	104	2215
108_4	"	36.0	294	2780	131_2	"	144.0	104	2215
108_5	"	36.0	296	2570	131_3	"	36.0	330	3990
109_1	"	144.0	186	2950	131_4	"	36.0	311	4260
109_2	"	144.6	186	3170	131_5	"	36.0	311	4310
110_1	"	144.8	210	2250	132_1	"	144.0	105	2220
110_2	"	144.8	210	2530	132_2	"	143.5	104	2020
111_1	1–1½–3	141.6	90	2400	133_1	"	144.0	95	2180
111_2	"	141.6	90	2470	133_2	"	144.0	95	2170
111_3	"	36.0	311	3470	136_1	"	144.0	103	2200
111_4	"	36.0	312	4100	136_2	"	144.0	103	2170
111_5	"	36.0	325	4170	136_3	"	36.0	328	3110
112_1	"	144.0	69	2600	136_4	"	36.0	335	3140
112_2	"	144.0	69	2600	136_5	"	36.0	338	3510
112_3	"	36.0	292	4000	137_1	"	143.5	99	1440
112_4	"	36.0	295	4730	137_2	"	144.0	99	1570
112_5	"	36.0	295	4340	138_1	"	144.0	72	1830
116_1	1–3–6	145.8	84	1440	138_2	"	144.0	77	2200
116_2	"	145.8	84	1460	138_3	"	36.0	298	2960
116_3	"	36.0	305	2480	138_4	"	36.0	299	2920
116_4	"	36.0	306	2490	138_5	"	36.0	305	2880
116_5	"	36.0	306	2260	141_1	"	143.0	105	2570
117_1	"	36.0	292	2490	141_2	"	144.0	105	2140
117_2	"	36.0	294	2360	142_1	"	145.0	92	1710
117_3	"	36.0	294	2520	142_2	"	144.0	92	1700
121_1	1–4–8	144.6	89	795	142_3	"	36.0	311	3340
121_2	"	144.0	96	990	142_4	"	36.0	312	3320
121_3	"	36.0	305	1410	142_5	"	36.0	333	3820
121_4	"	36.0	306	1630	143_1	"	144.0	93	2900
121_5	"	36.0	315	1410	143_2	"	142.8	93	2570

TABLE 6.—*Continued.*

No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.	No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.
146_1	1–2–4	144.0	100	1610	157_5	1-1½-3	36.0	291	3000
146_2	"	144.0	100	1580	158_1	"	144.8	37	2630
146_3	"	36.0	326	2890	158_2	"	144.0	37	2510
146_4	"	36.0	336	2680	171_1	1–2–4	144.0	101	2810
146_5	"	36.0	336	2930	171_2	"	144.6	101	2640
147_1	"	142.5	92	2420	176_1	"	143.3	106	1990
147_2	"	144.5	102	2280	176_2	"	142.8	99	1500
147_3	"	36.0	332	3830	177_1	"	145.2	92	1840
147_4	"	36.0	321	3660	177_2	"	145.9	92	1780
147_5	1–4–8	36.0	312	3700	178_1	"	144.0	69	1410
152_1	"	145.8	59	1100	178_2	"	144.6	69	1530
152_2	"	143.0	60	950	182_1	"	144.0	93	2720
152_3	"	36.0	280	1620	182_2	"	144.0	93	2800
152_4	"	36.0	286	1670	186_1	"	144.6	99	1450
152_5	"	36.0	289	1630	186_2	"	144.6	102	1410
153_1	"	143.5	245	960	187_1	"	144.6	86	1630
153_2	1–1½–3	145.2	245	985	187_2	"	144.6	86	1730
157_1	"	145.0	57	1880	187_3	"	36.0	296	3390
157_2	"	144.0	55	2130	187_4	"	36.0	308	3800
157_3	"	36.0	286	3400	187_5	"	36.0	300	3900
157_4	"	36.0	289	3060					

TABLE 7.

CYLINDER TESTS.

No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.	No.	Kind of Concrete	Area sq. in.	Age at Test days	Max. Load lb. per sq. in.
105	1–2–4	50.3	338	3480	138	1–2–4	50.3	298	2130
108	"	50.3	281	2670	142	"	50.3	311	2320
111	1–1½–3	50.3	309	2620	147	"	50.3	311	2910
112	"	50.3	292	3700	148	"	50.3	295	2280
116	1–3–6	50.3	305	1540	152	1–4–8	50.3	279	935
117	"	50.3	294	1260	157	1–1½–3	50.3	276	2470
121	1–4–8	50.3	306	785	172	1–2–4	50.3	315	3140
122	"	50.3	288	1140	181	"	50.3	319	2990
168	1–2–4	50.3	185	1790	186	"	50.3	318	1950

ments. These instruments were placed near the middle of the length of the column with the contact points resting sometimes

on the metal in the reinforced columns and sometimes on the concrete. The measurements taken are not as uniform or as trustworthy as could be desired, and the work must be considered as preliminary in character.

13. *General Method of Testing.*—A few days before testing, a plate similar to the base plate described above was set on a plaster of paris cushion on top of the column and served as the top bearing plate in the test. The top and bottom plates were connected with rods, and the columns were carried to the machine in a vertical position. The load was applied through a spherical bearing block which was carefully centered on the column. On the hooped columns, the load was applied in 20 000-lb. increments almost up to the maximum when 10 000-lb. increments were used. On the 9-in. columns the increments were made smaller. On the plain columns the load was run up from the start 10 000 lb. at a time. At each increment the machine was stopped while readings of the instruments were taken.

One 12-in. spiral-hooped column, No. 188, was tested under eccentric loading. A $\frac{1}{2}$-in. square mild steel bar was placed under the bearing blocks at top and bottom, each $1\frac{1}{4}$ in. off center, to obtain this condition.

III. Experimental Data and Discussion.

14. *Cube and Cylinder Test Data.*—Tables 6 and 7, page 18, give data of the tests of cubes and cylinders made from the same batch of concrete as the columns having the corresponding numbers. It was found impracticable to make many of these tests at the age at which the columns were tested and the tests were made at various ages as shown in the tables. The data do not generally allow a comparison to be made between the strength of the cubes and cylinders and that of the corresponding columns, though in the instances where the age of test makes the data comparable it is seen that the strength of the cube is generally greater than that of the column and that the strengths of the column and of the cylinder more nearly agree. The results for cubes from the same batch of concrete show great uniformity. The variety of age at test throws light upon the additional strength gained with time. The gain in strength is quite marked and is perhaps more than the gain usually expected.

15. *Column Test Data.*—Table 8, page 20, gives data on the age of test, maximum load and manner of failure of the plain concrete columns. Table 10, page 34, gives data of the tests of band-hooped columns and Table 12, page 36, the data of tests of spiral-hooped columns. The plain concrete columns will be considered first.

16. *Phenomena of the Tests of Plain Concrete Columns.*—The columns generally failed in the middle half of their length and the position and manner of failure showed general freedom from defects. No. 106, however, failed at the top with a load of 677 lb. per sq. in., and an examination showed that the concrete at this point was quite defective, lacking in solidity and strength, and that the column was far from being representative. The upper end of this

TABLE 8.

SUMMARY OF TESTS OF PLAIN CONCRETE COLUMNS.

Col. No.	Kind of Concrete	Maximum Load		Age at Test days	Manner of Failure
		Total pounds	lb. per sq. in.		
111	1–1½–3	240 000	2120	68	Diagonal shearing without warning
112	"	281 000	2480	62	" " " "
101	1–2–4	132 000	1165	58	Diagonal shearing
102	"	218 000	2000	69	General crushing
103	"	250 000	2210	65	" "
104	"	180 000	1590	64	" "
105	"	220 000	1945	62	Diagonal shearing
108	"	165 000	1460	72	General crushing
109	"	205 000	1810	64	Sudden diagonal shearing
116	1–3–6	108 000	955	61	Slow general crushing
117	"	126 000	1110	62	General crushing
121	1–4–8	65 000	575	63	Slow general crushing
122	"	65 000	575	63	" " "
110	1-2-4	218 000	1925	203	General crushing
128	"	209 000	1845	194	" "
129	"	200 000	1770	181	" "
163	"	303 000	2680	187	Sudden diagonal shearing
164	"	245 000	2160	187	Diagonal shearing
168	"	200 000	1770	201	General crushing
21	1-2-3⅜	381 500	2650	12 mo.	Diagonal shearing
22	"	389 000	2770	16 mo.	General crushing

column was afterwards cut off and another test made with results which showed that the remainder of the column was of normal composition. The results of No. 106 are not included in the data here given. In general, two somewhat distinct forms of failure were observed, (a) a failure which may be termed a diagonal shearing failure, and (b) a failure which may be termed simple compression. In the first the fracture was angular in nature, having the appearance of a diagonal shearing failure characteristic of the manner of failure in compression so common in brittle materials. In these failures the columns broke suddenly and without warning, some of them breaking even after the machine had been stopped and while no additional load was being applied. A loud report accompanied the failure. Column No. 112 failed suddenly and had given so little warning that the instruments had not been removed. In the second form of failure, here called "simple compression", the column shattered, cracking longitudinally for some distance, the cracks being quite well distributed over the faces of the column. In some of these, the failure was not noted until the weighed load began to decrease, and the position of the failure was not determined until the machine had produced a further shortening of the column. As will be seen in Table 7, nearly all the richer mixtures (1–1½–3 and 1–2–4) gave diagonal shearing failures, and the columns made with lean mixtures crushed and shattered throughout the whole fracture in what are here termed simple compression failures. Fig. 4 is a view of a typical diagonal shearing failure and Fig. 5 one of a simple compression failure. Of course, even the simple compression failures would be produced suddenly with a free dead load, but the action in the testing machine was quite distinct from the other. For both forms of failure, it may be stated in general that the approach of the ultimate strength of the columns might have been predicted from the increase in the rate of shortening and from the shape of the load-deformation diagram.

17. *Stress-deformation Diagrams for Plain Concrete Columns.*— In Fig. 16 to 20 following the text, the stress-deformation diagrams given represent the observed loads and the corresponding unit-deformations or shortenings for the columns tested. The method used in Bulletin No. 10 is followed. The ordinates represent the load or pressure in pounds per square inch on the col-

umn and the abscissas the unit-deformations or shortenings for a unit of length, as determined from the observed extensometer readings for the gauged length used. These values are taken from the average of the readings on the four faces of the column. The amount of deformation was calculated by using as the zero reading the extensometer reading taken at the original zero of load or load at which the first reading was taken; that is to say, the deformations shown are independent of any change or set which the load may have produced in the concrete. This is considered the proper method since one purpose of the determination of the shortening is for application to the case of a column with longitudinal steel reinforcement. This use of gross deformation and not of elastic deformation is discussed more at length in Bulletin No. 10.

18. *Strength of Plain Concrete Columns.*—In Table 8 in the column "Maximum Load" is given the ultimate load taken by the plain concrete columns. It will be noted that the richer mixtures give considerably higher strength than the leaner ones, the aver-

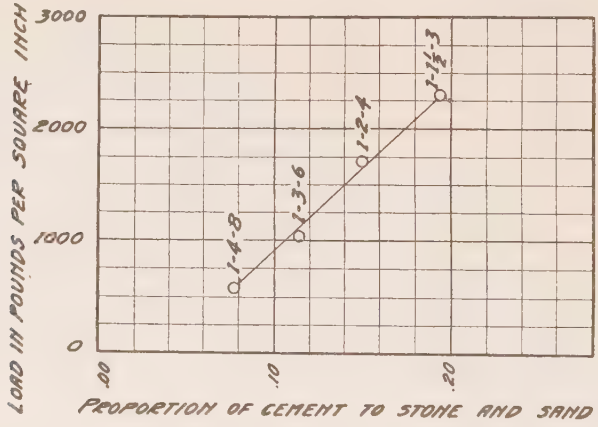

FIG. 6. RELATION BETWEEN STRENGTH OF CONCRETE COLUMNS AND AMOUNT OF CEMENT.

age at ages not far from 60 days being as follows: 1–1½–3 concrete, 2300 lb. per sq. in., 1–2–4 concrete, 1740 lb. per sq. in., 1–3–6 concrete, 1030 lb. per sq. in., and 1–4–8 concrete, 575 lb. per sq. in. In Fig. 6 this increase in strength with an increased proportion of cement is shown graphically, the weight of the cement being

TABLE 9.

PROPERTIES OF PLAIN CONCRETE COLUMNS.

Col. No.	Age at Test days	Kind of Concrete	Initial Modulus of Elasticity lb. per sq. in.	Abscissa of Vertex of Parabola	Maximum Stress lb. per sq. in.	
					Parabola	Observed
111	66	1–1½–3	2 800 000	.0015	2100	2120
112	62	1–1½–3	3 890 000	.0013	2530	2480
101	58	1–2–4	2 230 000	.0011	1225	1165
102	69	1–2–4	3 350 000	.0012	2010	2000
103	65	1–2–4	3 440 000	.0013	2240	2210
104	64	1–2–4	3 390 000	.00095	1610	1590
105*	62	1–2–4	2 660 000	.0014	1860	1945
108*	72	1–2–4	3 380 000	.0009	1520	1460
109	64	1–2–4	2 830 000	.0013	1840	1810
116	61	1–3–6	.....	.....	.....	955
117	62	1–3–6	.....	.....	.....	1110
121	63	1–4–8	.....	.....	.....	575
122	63	1–4–8	.....	.....	.....	575
106	286	1–2–4	.....	.....	.....	1460
110	203	1–2–4	3 270 000	.0012	1960	1925
128	194	1–2–4	3 140 000	.0012	1880	1845
129	181	1–2–4	3 600 000	.0010	1800	1770
163	187	1–2–4	3 000 000	.0017	2550	2680
164	187	1–2–4	3 100 000	.0014	2170	2160
168	201	1–2–4	2 770 000	.0013	1800	1770

*The columns marked (*) were made with Universal cement, all others with Chicago AA.

given in terms of the combined weights of sand and stone. The results for each mixture are the average of from two to five test columns. It is evident that cement is an effective reinforcing material and that as such it will compete very favorably with steel where compressive stresses only are to be taken. The addition of cement will give added strength quite out of proportion to the additional cost of the material. It is estimated that a mixture of 1–1½–3 would cost, say, 10% to 15% more than a 1–2–4 mixture, while the strength would be increased about 32%. A similar relation holds between the 1–3–6 and the 1–2–4 mixtures. The smaller sizes of columns necessary in the case of rich concrete may for the lower floors of a building effect a considerable saving in floor space, of itself an advantage.

A comparison of the strength of 1–2–4 concretes for the group tested at an age of about 60 days and for the group tested at an age of about 6 months shows that the strength of the concrete has increased in the interval something like 15%, though the variations in quality of stone and accidental variations make this comparison somewhat uncertain. The stone which is marked "soft" was much poorer in quality then that called "hard", and the effect of the quality of the stone used is quite apparent in the results of the tests.

It will be noted that the range of results is similar to that found in the plain concrete columns described in Bulletin No. 10. For the 1–2–4 concrete tested at an age of about 60 days the extreme range is 27% above and 34% below the average of the tests, and the average variation from the average strength is 12%. The strength of the 1–2–4 concrete was higher than the average for the 1906 tests, and this higher strength is due partly to the use of harder stone and partly to other changes in conditions. As in the 1906 tests, the ultimate strength was determinable with some accuracy by the location of the vertex of the parabola drawn to fit approximately the stress-deformation diagram. The loads corresponding to the vertex of the assumed parabola are given in Table 9 in the column headed "Parabola".

Attention has already been called to the fact that the strength of the columns is generally less than that of the cubes of the same mix and age and that the strengths of the columns and cylinders agree fairly well.

19. *Modulus of Elasticity.*—The treatment of modulus of elasticity of concrete will be the same as that given in Bulletin No. 10, Tests of Concrete and Reinforced Concrete Columns, Series of 1906. A discussion on the stress-deformation relation of concrete is also given in Bulletin No. 10. As there stated, modulus of elasticity is a term which has been used very loosely in connection with reinforced concrete. As a general property of materials, it is defined to be the ratio of the unit stress to the unit deformation within the elastic limit of the material. As applied in this way to materials having the property of proportionality of stress and deformation, the modulus of elasticity is a constant. For materials with a variable stress-deformation relation like concrete, the modulus of elasticity, as sometimes used, is still con-

sidered to be the ratio of the unit stress to the unit deformation regardless of the amount of the load carried relatively to the ultimate strength. As thus defined the modulus of elasticity of concrete will be a variable, and the value will decrease rapidly toward the ultimate strength of the concrete. For ordinary concretes, the stress-deformation relation may be considered to approach closely to a parabola with its vertex at the ultimate load, as was shown in Bulletin No. 10 and as is also apparent in the stress-deformation diagrams in this bulletin. The rate of change of stress and deformation at the beginning of loading is shown by the tangent to the parabola at the initial load and the value of the modulus of elasticity indicated by this tangent is called the initial modulus of elasticity. The values of the ratio between stress and deformation for the first one-third of the strength of the concrete do not vary much from this initial modulus of elasticity, but they are somewhat smaller for loads beyond this, the value of the ratio decreases rapidly and at the maximum load the ratio is only about half of the initial modulus of elasticity. It seems better to determine the initial modulus of elasticity, and then to obtain the relation between stress and deformation for other loads by means of a law which may be considered to express the changing relation. Formulas to show the relation between the stress and the deformation are given on page 27 of Bulletin No. 10, and a further discussion is also given on page 38 of that bulletin. It should be remembered that the value of the stress-deformation ratio for light loads on the concrete is somewhat smaller than the initial modulus of elasticity, so that if a straight-line stress-deformation relation (constant modulus of elasticity) is used for working loads, the value selected should be less than the initial modulus of elasticity. This must not be taken to mean that for relatively low loads the constant modulus of elasticity will give results which are greatly in error.

Comparison may then be made by means of the initial modulus of elasticity. The values of the initial modulus of elasticity for the several plain concrete columns tested are given in Table 8. The average for the 1-2-4 concrete at 60 days is 3 040 000 lb. per sq. in. This average is larger than that given in Bulletin No. 10 for the columns tested in 1906, which was 2 250 000. It was apparent from examination that the concrete in the 1907 tests was denser

than that in the 1906 tests and, besides, the stone used was generally a harder material. This may explain in part the higher value of the modulus of elasticity.

The effect of richness of concrete upon the value of modulus of elasticity is of interest. The values for the richer mixtures are given in Table 9. The leaner mixtures are somewhat indefinite, but it is evident that the addition of cement gives much stiffer concrete.

The results for 1–2–4 columns tested at an age of 181 to 203 days give about the same value for the initial modulus of elasticity as those at 60 days.

20. *Poisson's Ratio.*—When a load is applied to a column or other test piece in compression, the column shortens longitudinally and at the same time expands laterally or transversely. The ratio between the lateral unit-deformation involved in this lateral expansion and the unit-deformation or the longitudinal shortening of the piece is termed "Poisson's Ratio". Poisson's ratio is of interest in connection with the effect of steel hoops in hooped concrete columns for loads within the compressive strength of plain concrete. It is also of interest in studying the relation between tensile and shearing stresses and the compressive strength of brittle materials. Little definite information on the value of Poisson's ratio for concrete is available. The values most frequently quoted are 0.25 and 0.3.

In the tests of the plain concrete columns, observations were made to determine a value for Poisson's ratio. Necessarily the apparatus and methods used were experimental in character. Refinement of measurement is essential, and any lack of rigidity in the apparatus or inertia of parts will interfere with the accuracy of the data. The results show inherent defects, but their general trend may serve to throw light on this property of concrete. Improvements in the instruments have now been made, and other tests are now under way from which a more definite value of Poisson's ratio for concrete may be expected.

From a study of the observations obtained, it would appear that Poisson's ratio for 1–2–4 concrete 60 days old for the lower loads is probably $\frac{1}{10}$ to $\frac{1}{6}$ and that near the ultimate load the ratio increases, probably reaching $\frac{1}{4}$. Tests were made with better apparatus on concrete columns 6 months old. For Column No. 163

the value is about 0.18 and for Column No.164, 0.11 to 0.18. Still later tests give values considerably lower than these. The amount of the lateral deformations is shown in Fig. 20. It should be remembered that this concrete set in air.

21. *Repetitive Loading of Plain Concrete Columns.*—In most of the tests the column was loaded progressively to failure; that is, without releasing the load during the test. When a load is applied in any amount and released, the load then reapplied and this released, and this application and release of the load repeated, the longitudinal deformation of the concrete generally increases, and there is also an increase in the set when the load is released. This statement is true even for low loads. For the smaller loads there is a limit to the increase in deformation due to the repetition of the load. The total deformation and the resulting set soon become constant or nearly constant. For the higher loads the deformation and set continue to increase during a larger and larger number of applications, and failure will occur under repetitive

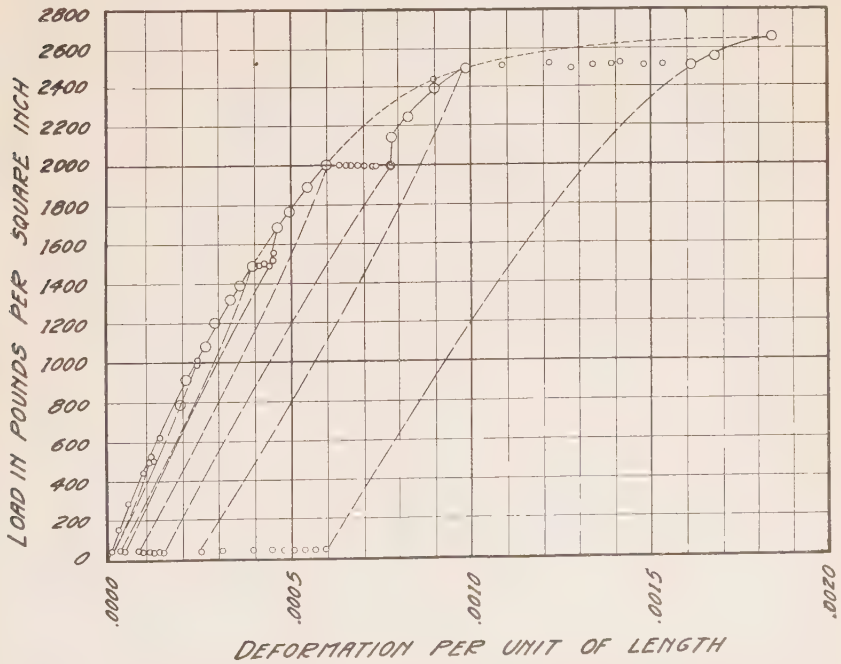

FIG. 7. STRESS-DEFORMATION DIAGRAM FOR CONCRETE COLUMN NO. 21.

loading at a load below the maximum strength for a single application. The number of repetitions necessary to produce a limit to the increase, and the amount of the increase of deformation, will of course vary with the quality and the age of the concrete. The resulting net deformation is generally called the elastic deformation, though it must be borne in mind that the proper-

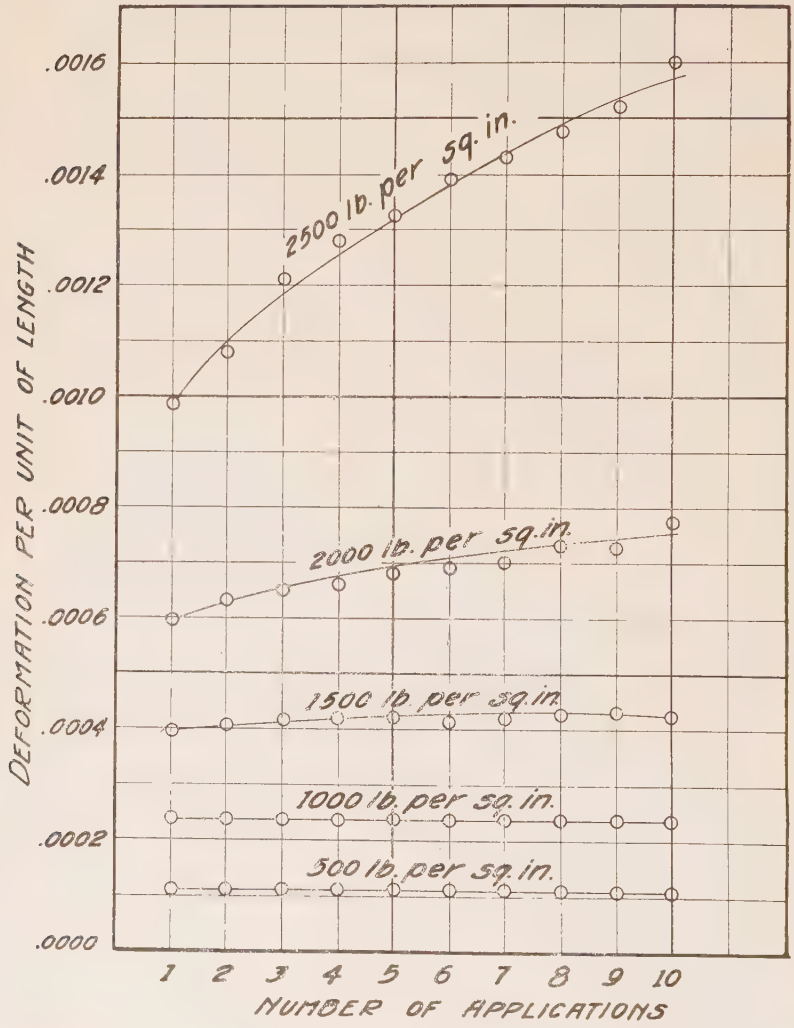

FIG. 8. CHANGE IN DEFORMATION IN CONCRETE COLUMN NO. 21.

ties of concrete found by using the elastic deformation are not generally applicable in the analysis of reinforced beams and columns. Whenever, after the repetition of a given load, the load is increased, the stress-deformation curve at once rises until it joins the general direction of the stress-deformation curve for a single application of the increased load. If the points for the last repetition of each load be joined, a curve having the same general form as the curve for single applications will be obtained, but with greater deformations. This general effect is illustrated graphically in Fig. 11, page 40, of Bulletin No. 10.

Columns No. 21 and 22 were tested by applying and releasing a load 10 times, then applying a greater load 10 times, and so on to the failure of column. The change in deformation and in set is shown in Fig. 7. The increase of the deformations with the repetition of the several loads for Column No. 21 is shown in Fig. 8. It will be noted that for the smaller loads the deformation increases very slowly after a few repetitions. For the higher loads the deformation continues to increase. Evidently at the upper loads a large number of repetitions would finally produce failure. The change in the set follows the same law.

22. *Phenomena of Tests of Hooped Columns.*—The general phenomena accompanying the tests of hooped columns are as follows: The earlier part of the test is much the same as for plain concrete columns. At a load equal to that which would cause failure in a plain concrete column, or a little above, the concrete over the spacing bars begins to scale, and this is soon followed with a scaling and shelling off of the surface of the column over the hoops everywhere. With added increments of load beyond the limit of strength of plain concrete the amount of shortening increases rapidly and the column expands or bulges laterally correspondingly rapidly. The maximum load in some cases may not be reached until the hooping has been stressed to its elastic limit, and in other cases this load may be reached without developing the elastic limit of the hooping, especially when large lateral deflection of the column and general distortion from the original shape occur. The lateral deflection of the column from a straight line is apparent before the maximum load carried is reached —in many cases considerably before. After the maximum load is passed, the lateral deflection rapidly increases. During the

last stage of the test the column usually forms a curve having the characteristics of the figure shown in text books for a column with fixed ends. Fig. 9 and 10 give views of hooped columns after the maximum load has been passed. Some columns finally bent out of line as much as 4 or 5 inches, although the load then carried was only a small proportion of the maximum and this condition could not obtain under a dead load.

There was a variation in the time of the first noticeable lateral bending in the tests. Generally speaking, in the case of companion columns, this bowing was perceptible first in the one which gave the lowest maximum strength. This load at which bowing became apparent ranged from 60% to 85% of the maximum load of the column in the case of the band-hooped columns and from 70% to 95% in the case of the spiral-hooped columns, though in the latter the average was nearly 90%. However, it was evident that slight bending began earlier than this. The amount of this bowing at the time of the maximum load was not large, but its presence, or the presence of the conditions which caused the bending, must have had much to do with the final failure of the column. In most of the tests, after passing the maximum load, the bending increased rapidly under loads smaller than the maximum and the column can not be considered to have failed by direct crushing.

The amount of shortening in the hooped column at the maximum load, as compared with plain concrete columns at their maximum load, was very great. The total amount of shortening in a hooped column 10 ft. long ranged from $\frac{3}{4}$ in. to $1\frac{3}{4}$ in., and the corresponding longitudinal unit-deformation was from 0.006 to 0.015. This is six to twelve times the shortening in a plain concrete column at its maximum load. The amount of set produced in the column is very great. In Column No. 173, as shown in Fig. 24, for a load well below the ultimate strength of the column, this set was 0.0025 per unit of length, or over two thirds of the total unit shortening of the column at the first application of the load.

The purpose of reinforcing concrete columns with hooping is to restrain the concrete laterally and to add to the strength of the column by preventing the disintegration of the concrete when that limit of lateral expansion has been reached for which in plain concrete rupture is produced by the internal tensile or shearing

stresses developed. A little study will show that the effect of the hooping for loads below the ultimate strength of plain concrete must be relatively small. Consider that the load on the column is 800 lb. per sq. in., about one-half of the ultimate strength of the plain columns for the 1–2–4 columns 60 days old of the tests under consideration. The amount of shortening for a concrete column at half its ultimate load may be considered to be 0.0003 units per unit of length (inches per inch of length). If we consider that Poisson's ratio for this concrete is $\frac{1}{4}$, the lateral expansion for the column will be 0.0000375. Even if we consider that the steel hooping would receive this unit deformation the stress in the hooping would be only 1100 lb. per sq. in., and reasoning from this it seems evident that the effect of the hooping for lateral deformations of this amount is very slight. A comparison of the stress-deformation diagrams for plain columns and for hooped columns shown in succeeding pages bears this out. It seems probable that voids left in the concrete may interfere with early pressure on the hooping. For columns reinforced with steel rods placed longitudinally in the column, with a load on the column giving a shortening of 0.0003, the stress in the steel would be 9000 lb. per sq. in., a much larger amount proportionally. For hooped columns, if the value of Poisson's ratio remains at $\frac{1}{4}$ for higher loads, the stress in the hooping would increase somewhat proportionally. When the longitudinal shortening became 0.001, the steel in the hoops would be stressed 3750 lb. per sq. in. If the value of Poisson's ratio increases as the load at which plain concrete would fail is approached, considering it to be 0.25 at this point and considering also that the longitudinal shortening of the column at this load is 0.001, the stress in the hooping would become 7500 lb. per sq. in. Beyond the load which would produce failure in unrestrained concrete the conditions in the columns change materially, as is shown in succeeding pages, and the hooping receives additional stress rapidly and its function in maintaining the integrity of the concrete is important. It is to be expected that if other conditions were maintained the limit of strength of the column would be reached when the hoops become stressed to their yield point. It is also to be expected that if the hooping becomes pressed into the concrete, or if by reason of lack of homogeneity or other cause the column bends or deflects laterally and the pres-

sure is unevenly distributed, the maximum load may be reached before the hoops can become stressed to their yield point. The tests indicate that such conditions exist.

23. *Stress-deformation Diagram for Hooped Columns.*—Stress-deformation diagrams for hooped columns are given in Fig. 21 to 25 at the end of the text. The same general notation is followed in these diagrams as is used in the diagrams for plain concrete columns. Fig. 11, page 32, shows Column No. 138, which was reinforced with steel bands and which may be considered to be typical of the banded columns. Fig. 12, page 33, shows Column No. 182 which was reinforced with spiral hooping and which may be considered to be typical of the spiral-hooped columns. An examination of all the diagrams shows that the stress-deformation diagram given by hooped columns is much the same as that given by plain concrete columns up to that load which would cause failure in a plain concrete column and up to an amount of shortening in the column, of, say, 0.001 or 0.0015, unit-deformation. That is to say, the comparison shows a very close relation between loads and shortenings for the two classes of columns up to the point of

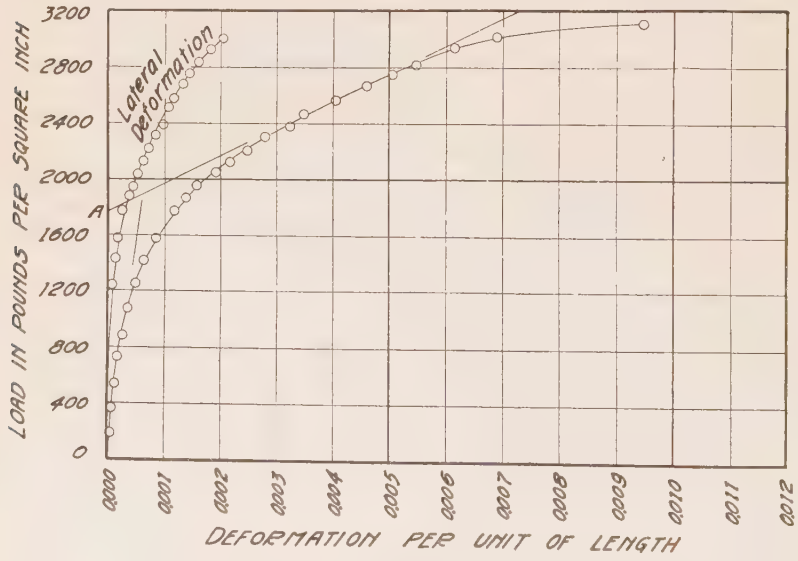

FIG. 11. STRESS-DEFORMATION DIAGRAM FOR BAND-HOOPED COLUMN NO. 138.

FIG. 9. BAND-HOOPED COLUMN NO. 136 AFTER TEST.

FIG. 10. SPIRAL-HOOPED COLUMN NO. 181 AFTER TEST

failure for plain concrete columns. For loads greater than the ultimate strength of plain concrete, the shortening of the hooped columns increases very rapidly, the ratio of the increment of load to the increment of longitudinal deformation (unit shortening) being only something like 100 000 to 200 000, varying according to the amount of reinforcement. The stiffness of a hooped column for the added load after the hooping has come into action is therefore very small as compared with that of a plain concrete column within its limit of strength. The stress-deformation line at this stage of the test is fairly straight and it gradually swings to a horizontal position as the maximum load of the column is reached. Beyond this horizontal portion, if the test is continued, the load falls off and the shortening is continued as the column gradually bends out of line. If the straight line which fits the part of the stress-deformation curve where the hooping is effective be produced back to the axis of ordinates, (as shown in Fig. 11 and 12)

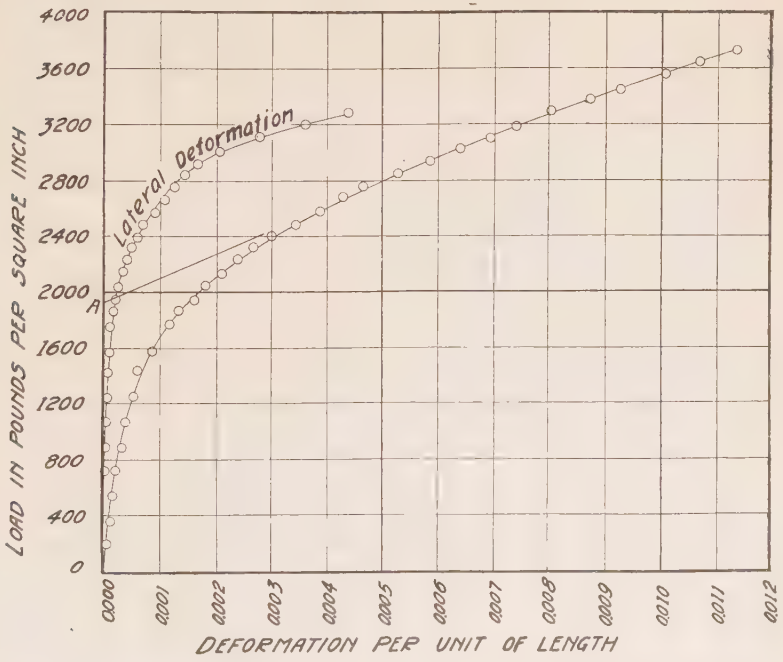

FIG. 12. STRESS-DEFORMATION DIAGRAM FOR SPIRAL-HOOPED COLUMN NO. 182.

the intersection agrees closely with the load which produces a longitudinal shortening of 0.001 or 0.0015. As stated later, there is a good indication that this agrees closely with the strength that the column would have without hooping. The agreement of these values is noteworthy. The general slope of the stress-deformation curve above this point varies with the amount of the

TABLE 10.

SUMMARY OF TESTS OF BAND-HOOPED COLUMNS

Col. No.	Kind of Concrete	Reinforcement			Maximum Load		Age at Test days
		Size and Spacing of Bands	Section of Band sq. in.	Per cent	Total pounds	lb. per sq. in.	
		12-in. Columns.					
131	1–2–4	No. 16, 2 in. cc.	.065	1.08	270 000	2384	60
132	1–2–4	No. 16, 2 in. cc.	.065	1.08	243 000	2150	57
133	1–2–4	No. 16, 2 in. cc.	.063	1.05	247 000	2182	59
136	1–2–4	No. 12, 2 in. cc.	.125	2.08	323 000	2860	63
137	1–2–4	No. 12, 2 in. cc.	.124	2.07	300 500	2660	69
138	1–2–4	No. 12, 2 in. cc.	.127	2.12	351 000	3110	60
146	1–2–4	No. 8, 2 in. cc.	.193	3.22	339 000	3000	60
147	1–2–4	No. 8, 2 in. cc.	.192	3.20	420 000	3715	66
148	1–2–4	No. 8, 2 in. cc.	.192	3.20	327 000	2890	63
143	1–2–4	No. 12, 3 in. cc.	.125	1.39	309 000	2735	60
141	1–2–4	No. 12, 4 in. cc.	.122	1.02	256 000	2275	59
142	1–2–4	No. 12, 4 in. cc.	.122	1.02	246 000	2178	66
		9-in. Columns					
152	1–4–8	No. 16, 2 in. cc.	.061	1.35	86 000	1345	70
153	1–4–8	No. 16, 2 in. cc.	.063	1.41	80 000	1260	56
151	1–2–4	No. 16, 2 in. cc.	.066	1.47	137 000	2140	65
156	1–2–4	No. 12, 2 in. cc.	.123	2.73	190 000	2970	67
157	1–1½–3	No. 12, 2 in. cc.	.132	2.94	228 000	3561	67
158	1–1½–3	No. 12, 2 in. cc.	.126	2.80	234 000	3685	81

reinforcement. It will be seen that there is some difference in the amount of deformation for columns hooped with bands and with spirals. This is discussed in a succeeding paragraph. On some of the figures the lateral or transverse deformation diagram is given. It must be borne in mind that the values shown by these

TABLE 11.

PROPERTIES OF BAND-HOOPED COLUMNS

Col. No.	Per cent Rein.	Strength of Column in lb. per sq. in.		Additional Strength per 1 % of Reinforcement lb. per sq. in.	ϵ'	Initial Modulus of Elasticity lb. per sq. in.	ϵ''	Ratio of Increments
		Concrete from Diagram	Maximum					
12-in. Columns								
131	1.085	1600	2384	725	.0012	2 670 000	.004	196 000
132	1.085	1400	2150	690	.0012	2 340 000	.005	150 000
133	1.050	1450	2182	695	.0011	2 640 000	.0045	163 000
136	2.081	1600	2860	605	.0011	2 910 000	.006	210 000
137	2.071	1200	2660	705	.0012	2 000 000	.0085	172 000
138	2.12	1750	3110	640	.0012	2 920 000	.007	194 000
146	3.22	1250	3000	545	.0011	2 280 000	.0103	170 000
147	3.20	1650	3715	645	.0013	2 670 000	.009	230 000
148	3.20	1900	2890	310	.0012	3 160 000	.0055	180 000
143	1.39	1650	2735	780	.0011	3 000 000	.0055	197 000
141	1.02	1600	2275	660	.0012	2 670 000	.0045	150 000
142	1.02	1500	2178	665	.0010	3 000 000	.0045	150 000
9-in. Columns								
152*	1.35	650	1345	515	.0012	1 080 000	.008	87 000
153*	1.41	600	1260	470	.0010	1 200 000	.008	83 000
151	1.47	1200	2140	640	.0010	2 400 000	.004	235 000
156	2.73	1600	2970	500	.0012	2 670 000	.0063	218 000
157*	2.94	1950	3561	550	.0011	3 550 000	.006	269 000
158*	2.80	2150	3685	550	.0011	3 910 000	.0058	265 000

*All the columns were of 1-2-4 concrete, except 152 and 153, which were 1-4-8, and 157 and 158, which were 1-1½-3.

lines are not very accurate, but they serve to give an idea of the relation existing. Attention is called to the fact that these deformations are all based upon the original length of the column, since that is the condition which would exist in construction, and that the effect of set is not here considered.

TABLE 12.

SUMMARY OF TESTS OF SPIRAL-HOOPED COLUMNS.

In all cases pitch of spiral was 1 inch.

Col. No.	Kind of Concrete	Reinforcement			Maximum Load		Age at Test days
		Size of Wire	Section of Wire sq. in.	Per cent	Total pounds	lb. per sq. in.	
			High-carbon steel				
171	1–2–4	No. 7	.025	0.85	283 000	2503	56
172	1–2–4	No. 7	.025	0.85	283 000	2506	63
173*	1–2–4	No. 7	.024	0.82	277 000	2000*	57
181	1–2–4	¼-in.	.052	1.73	307 000	2718	57
182	1–2–4	¼-in.	.050	1.67	429 000	3800	60
183	1–2–4	¼-in.	.050	1.68	428 000	3793	63
			Mild steel				
176	1–2–4	No. 7	.025	0.84	235 000	2080	60
177	1–2–4	No. 7	.025	0.85	249 000	2203	64
178	1–2–4	No. 7	.025	0.84	251 000	2220	61
186	1–2–4	¼-in.	.049	1.64	234 000	2068	58
187	1–2–4	¼-in.	.051	1.71	385 000	3404	62
188	1–2–4	¼-in.	.048	1.61	147 000	.....	59

*This column was not tested to destruction as a hooped column, but after a load of 2000 lb. per sq. in. had been applied, the spiral was stripped from the concrete and the resulting plain concrete core tested as noted in the text.

24. *Modulus of Elasticity and Compressive Deformations.*—If the hooping is effective in any marked degree at the beginning of the test it may be expected that the initial modulus of elasticity will be greater for hooped columns than for plain concrete columns and that the stress-deformation diagram will show that the col-

TABLE 13.

PROPERTIES OF SPIRAL-HOOPED COLUMNS

All the columns were made of 1–2–4 concrete.

Col. No.	Per cent Rein.	Strength of Column, lb. per sq. in.		Additional Strength per 1% Reinforcement lb. per sq. in.	ϵ'	Initial Modulus of Elasticity lb. per sq. in.	ϵ''	Ratio of Increments
		Concrete from Diagram	Maximum					
				High-carbon Steel				
171	0.85	1600	2503	1060	.0012	2 670 000	.005	180 000
172	0.85	1700	2506	950	.0012	2 840 000	.005	161 000
173	0.82	1400	2010	.....	.0010	2 800 000	.....	.....
181	1.73	1400	2718	760	.0014	2 000 000	.010	127 000
182	1.67	1900	3800	1140	.0014	2 720 000	.011	173 000
183	1.68	1900	3793	1130	.0015	2 540 000	.011	172 090
				Mild Steel				
176	0.84	1350	2080	870	.0012	2 250 000	.0045	162 000
177	0.85	1500	2203	830	.0012	2 500 000	.0045	156 000
178	0.84	1450	2220	920	.0012	2 420 000	.005	154 000
186	1.64	1150	2068	560	.0012	1 920 000	.008	115 000
187	1.71	1800	3404	940	.0011	3 280 000	.0065	247 000
188*	.....	.....	.....	.....	.....	.....	.....	.....

*Not included in this table on account of being loaded eccentrically. (See p. 45).

umn is stiffer for loads up to the ordinary compressive strength of concrete. It is worth while to make an examination of the portion of the stress-deformation curve for the first stage of the test. To do this the point where the first marked change occurs in the direction of the line from that for plain concrete, or which corresponds with the ultimate longitudinal deformation found in plain concrete columns (ranging from 0.001 to 0.0015) indicated in the tables by ϵ', is determined. A study of the diagrams makes it seem probable that the load causing this shortening corresponds closely with the ultimate strength of the concrete of the column. Tables 11 and 13, p. 35 and 37, give the results found. The

variation in these values is not greater than that found in plain concrete columns, but the average strength is somewhat less than the average for the plain concrete columns of the same mix and age. The tests of cubes and cylinders from the same mix, so far as these were made, confirm the belief that the values thus found are representative of the strength of the concrete. It is interesting to note that the load thus obtained agrees fairly well with that found by taking the straight portion of the stress-deformation curve as it averages beyond the elbow and extending it backward to an intersection with the axis of ordinates

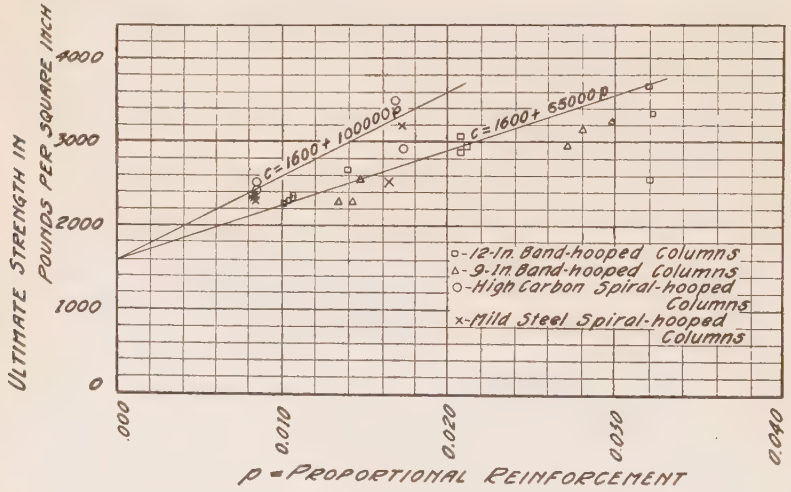

FIG. 13. ULTIMATE STRENGTH OF HOOPED COLUMNS.

(A in Fig. 11 and 12). After determining the critical point above described, the calculation of the initial modulus of elasticity may be made by the method used for plain concrete columns. These results are also given in Tables 11 and 13, p. 35 and 37.

The initial modulus of elasticity for the first stage obtained in this way averages about 2 610 000 lb. per sq. in. for the hooped columns made of 1–2–4 concrete. The average initial modulus of elasticity for the corresponding plain concrete columns is 3 040 000 lb. per sq. in. As a matter of fact the determination of this initial modulus of elasticity for hooped columns is subject to variation of judgment, but it appears then that the hooped columns are less stiff through the first stage than are the plain concrete columns.

Using the parabolic relation to determine points for lower loads than the assumed ultimate strength of the concrete (the method found satisfactory for the plain concrete), the calculated points so obtained show a very close agreement with the observed stress-deformation curve. No reason is known for the apparent decrease in stiffness, though the presence of the hooping may affect the fabrication of the columns or their shrinkage in setting and the density of the concrete. At any rate, it seems evident that there

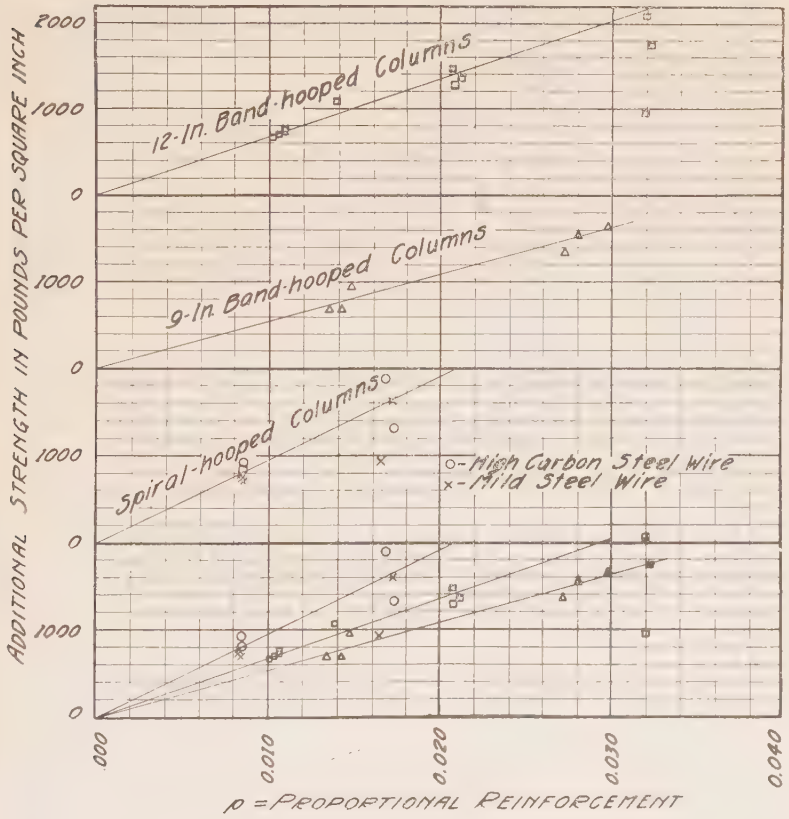

FIG. 14. ADDITIONAL STRENGTH DUE TO HOOPING.

must be very little restraint by the hoops through this stage. An analytical treatment of the stresses which may be developed in hoops, especially with the low value of Poisson's ratio then acting, leads to similar conclusions.

When loads are applied which give shortenings larger than 0.001 to 0.0015, the second stage begins and a considerable stress is brought into action in the hooping. As the load is added the restraint of the hooping becomes marked and there is a very rapid increase in the amount of shortening produced by the added load. The stress-deformation diagram, through this stage of the test, approximates a straight line for the banded columns. For those with spiral reinforcement only a small portion is straight, but the main part of the curve has one general direction. The ratio of the increment of load to the increment of deformation, however, is comparatively small. It is well to compare this ratio with the modulus of elasticity of concrete. The values for this ratio, determined from the stress-deformation diagrams are given in Tables 11 and 13, in the columns headed "Ratio of Increments", ϵ'' being the unit-deformation corresponding to the maximum load. The average value for this ratio for columns with band reinforcement is 184 000; for those with spiral reinforcement, 165 000. This ratio is very much less than the modulus of elasticity of concrete. The values for Columns No. 152 and 153 show that the ratio becomes much smaller for leaner concrete; but those for Columns No. 157 and 158 indicate that it is higher for the richer mixture. Attention is called to the fact that the deformations used refer to the original length and not to the length after the set is deducted.

It will be seen that the amount of shortening in the column before the maximum load is reached depends upon the amount of reinforcement, and that it is more for columns with spiral reinforcement than for columns reinforced with bands. The average value for the banded columns given in terms of the excess of the unit shortening of the hooped column over the shortening for a plain concrete column per 1% of reinforcement ranges from 0.0031 to 0.0059 for the columns with spiral reinforcement, and 0.0013 to 0.0035 for columns reinforced with bands. Attention should be called to the fact that the columns reinforced with spiral will sustain higher loads, as well as shorten more, than the columns reinforced with bands. It is not known whether the concrete bulged out more between the spirals than between the hoops, but this condition seems probable.

25. *Strength of Hooped Columns.*—In Table 10 are given the

loads carried by the columns reinforced with bands, and in Table 12 those with spiral reinforcement. Fig. 13 shows the results graphically. It is evident that hooping adds strength and that the excess of strength over that of plain concrete columns is proportional to the amount of reinforcement. The full effect of the hooping is not clearly shown in Fig. 13 since the concrete in each column has its individual strength. A better comparison is made by first estimating the strength of the concrete for each column as found in "23. Modulus of Elasticity and Compressive Deformations". If the increase in strength over that of a plain concrete column be divided by the per cent of reinforcement, the additional strength per 1% of reinforcement is obtained. This is shown in Table 11 and Table 13 in the columns headed "Additional strength per 1% of reinforcement". It is also shown graphically in Fig. 14. The strength assumed for the concrete of the individual columns involves errors of judgment, even if the method of determining it is considered to be a proper one, but a careful study of the diagrams shows that the variation due to judgment may not, for average of the results, amount to more than 50 lb. per sq. in. above the values assumed, and this variation will not seriously affect the values found for the added strength of the column due to hooping. The strength of the 1-2-4 concrete given for the hooped columns is less than the average for the 1-2-4 concrete columns. Part of this may possibly be due to difference in materials and part to the effect of the presence of the hooping upon the density of the concrete.

For the columns reinforced with bands there is a fair agreement in the value of this additional strength per 1% of reinforcement for all the 12-in. columns regardless of the amount of reinforcement. The average is 669 lb. per sq. in., if we except Column No. 148 which appears by the stress-deformation diagram and other data to be erratic, probably the result of poor concrete or of defective hooping. The 9-in. band-hooped columns are also fairly uniform, the average additional strength being 537 lb. per sq. in. per 1% of reinforcement. It is noticeable that the additional strength per 1% of reinforcement in these 9-in. columns is much the same for the 1-4-8, 1-2-4 and 1-1½-3 concrete, and that the difference in the strength of these columns seems to show up mainly through a difference in the strength of the con-

crete itself. It should be recalled, however, that the hooped column with 1–4–8 concrete is much less stiff through the second stage than is that with 1–1½–3 concrete. The reader should also keep in mind the low degree of stiffness in hooped columns, as discussed elsewhere.

For spiral hooping there is a greater additional strength per 1% of reinforcement, and also more variation in the results. Omitting No. 186 which seems to be poor material, the average additional strength per 1% of reinforcement for these columns is 955 lb. per sq. in., which is about 50% greater than the additional strength found in the columns reinforced with bands. There seems to be little difference between the strength given by the high-carbon wire and the mild-steel wire, especially when it is remembered that the No. 7 high-carbon wire has a yield point of 60 000 lb. per sq. in. and the ¼-in. high-carbon steel wire of 110 000 lb. per sq. in. The results with the mild-steel wire are somewhat lower, but not so much as might be expected when the difference in elastic limit is taken into consideration.

What the cause is of the greater added strength per 1% of reinforcement with the spirals than with the bands, cannot be stated. The observations indicate that the lateral deformation of the concrete between hoops at the maximum load is greater in columns with spirals than in those with bands. Considering this and also that the columns with spirals compress more before final failure, as shown by the stress-deformation diagrams, and that the stress-deformation curve bends out of a straight line into a long curve, it would seem possible that the concrete squeezes out between the hooping, so to speak, in a larger proportional degree with the spiral reinforcement and thus that the spiral hooping was not stressed up to its elastic limit. The measurement of the stretch in the bands indicates that in some of the columns this material reaches its elastic limit. It seems evident that the hoops are stretched more at their edges than at their center. It is likely that in the placing together of the band reinforcement by the workmen in the laboratory, the structure formed was more uneven and less perfect than was the spiral reinforcement. The whole question is complicated by the bulging or lateral deflection of the columns, and this appeared at a relatively earlier stage with the bands than with the spirals. A study of the read-

ings of the individual extensometers indicates that there may be quite a difference in the amount of shortening at points around the column before lateral deflection of the column is noticeable. As this part of the problem was unexpected, the observations were not made carefully enough to state definite results.

A general form of formula for hooped columns may be given as follows:

$$c = c' + pc''$$

where c is the maximum strength of the hooped column, c' is the strength of the plain concrete column, c'' is a coefficient for the hooping and p is the ratio of reinforcement found by dividing the total volume of the hoops by the volume of the core within the hoops, in other words, by considering the hooping to be distributed as a thin cylinder continuously along the hooped column.

The values given by the writer in the Proceedings of the American Society for Testing Materials, for 1907, are not far from the average results for the 12-in. columns. Using these, the expression for the columns reinforced with bands may be given as

$$c = 1600 + 65\,000\,p$$

and that for the spiral as

$$c = 1600 + 100\,000\,p$$

The latter equation may be considered to give values too large for the mild-steel wire and smaller than the average for high-carbon steel wire. Considerable variation from these values may be expected.

26. *Lateral Pressure and Stress in Hooping.*—The expansion of the column under load is accompanied by a lateral pressure and this pressure develops stresses in the hooping. It seems from the stress-deformation diagrams that the lateral pressure is proportional to the excess of the load over the strength of the plain concrete. The observations made on the expansion of the bands also indicate that the stress in the hooping is proportional to the increments of load beyond the load which plain concrete would resist. No analytical treatment of the stresses produced by the concrete as it expands will be gone into, but it may be worth while to see what stresses are possibly developed in the hooping and what the relation between the longitudinal compressive stress and the lateral expansive stress may be. It is evident from the tests that many of the band-hooped columns failed by the bands becoming stressed

beyond the yield point of the metal. This gives a clue to the amount of stress existing.

The following nomenclature is used: t' the thickness of band, b the breadth of one band, a the distance from center to center of bands or spirals, t'' the diameter of the wire of the spiral, p the ratio of the volume of the hooping to the volume of the core or

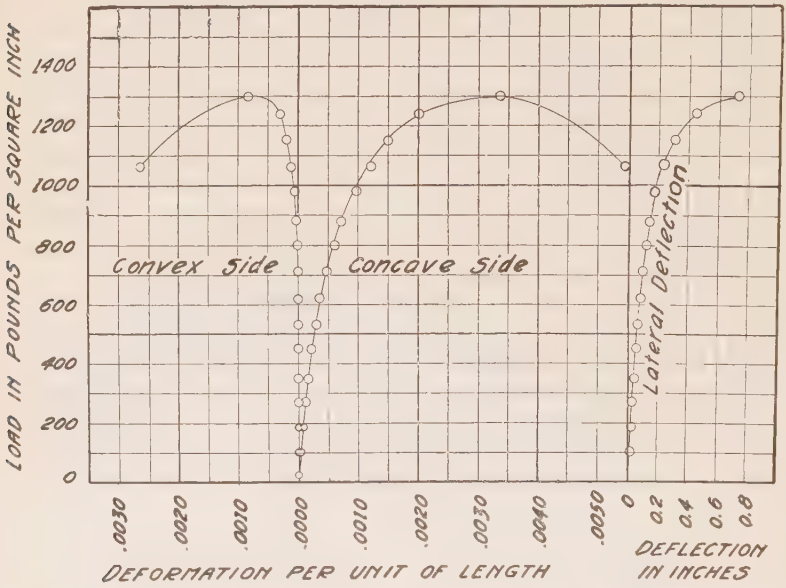

FIG. 15. CURVES FOR SPIRAL-HOOPED COLUMN NO. 188, ECCENTRICALLY LOADED.

part of the column within the hooping, and d the diameter of the hooped core. Then the equivalent thickness of an enveloping cylinder of metal is for the band-hooped columns

$$t = \frac{b}{a} t'$$

and for the spiral hooping

$$t = \frac{t''^2}{a\,d^2}.$$

For either case we have

$$t = \frac{p\,d}{4}.$$

If we consider by analogy that the lateral pressure is caused

by a lateral pressure in the concrete similar to hydrostatic pressure, using P as this lateral pressure in pounds per square inch, we shall have from the ordinary formula for thin cylinders

$$P = \frac{pf}{2}$$

where f is the stress in the hooping in lb. per sq. in. If now the stress in the hoops for the maximum load on the column be taken at the yield point of the material, 48 000 lb. per sq. in., this gives

$$P = 24\,000$$

or 240 lb. per sq. in. equivalent lateral pressure for each 1% of hooping. The formula already used considers that 650 lb. per sq. in. added compressive strength is given for 1% of band hooping. From this it would seem that the ratio of the equivalent lateral pressure to the added strength given by the reinforcement in the case of the band-hooped columns is 0.36. The probability that many of the columns failed by lateral bending and without stressing the bands to the elastic limit should be kept in mind in this connection. While this may not be an accurate value it may give some indication of the ratio between the lateral stress and the longitudinal stress in such columns.

27. *Condition of Hooped Concrete after Testing.*—An experiment of some interest was made by first loading a hooped column and then stripping the wire from it and testing the naked column. Column No. 173 (No. 7 wire, 0.82% reinforcement) was loaded to 2000 lb. per sq. in. with a resulting unit shortening of 0.0036. The load was then released and a set of 0.0025 was found. A load of 1945 lb. per sq. in. was then applied when the unit shortening became 0.0041. Upon the releasing of this load the set was found to be 0.0027. The spiral was then stripped from the column without taking it from the machine and a load was applied to the naked column. The column failed at 1080 lb. per sq. in. The indications of the stress-deformation diagram are that the plain concrete had a strength originally of about 1400 lb. per sq. in. It seemed likely that the stripped column sustained as much as 75% of its original strength as plain concrete. A comparison with the other columns and the form of the stress-deformation diagram indicates that the hooped column might have held 2400 lb. per sq. in. before failure.

28. *Eccentric Loading.*—Column No. 188 was tested with the

pressure applied both top and bottom along a line of $1\frac{1}{4}$ in. away from the center of the column. This column was reinforced with $\frac{1}{4}$-in. wire giving 1.62% of reinforcement. The phenomena accompanying the test were similar to those for central loading except that the lateral deflection was apparent at an earlier load and increased rapidly. The lateral deflection amounted to 0.72 inch when the maximum load was reached. Fig. 15 shows the deformations on two sides of the column and also the amount of lateral deflection. When the eccentricity of the load is considered and also the amount of eccentricity due to the lateral bending, the final loading carried agrees very closely with what would be expected with a homogeneous column under the same conditions. It is evidently not necessary to have the internal pressure uniformly distributed over the cross section of the column in order to utilize the strength of the hooping as would be the case if the internal pressure were considered to act in a manner similar to hydrostatic pressure. The column tested showed considerable toughness, a property which would not be present in plain concrete eccentrically loaded.

29. *Summary.*—The writer feels that there is yet much to be learned about concrete and reinforced concrete columns. After making the tests described he has had to give up several preconceived notions of the action of hooped concrete, notions based upon meager statements of tests and analyses made from assumed conditions. The tests here given are not altogether conclusive, but they may help to clear up uncertainties and give an idea of the action of concrete when restrained by hooping. The tests on plain concrete columns also bring out instructive results. The following observations on the tests may be made:

1. The average ultimate strength of the plain concrete columns at ages not far from 60 days is as follows: 1–$1\frac{1}{2}$–3 concrete, 2300 lb. per sq. in.; 1–2–4 concrete, 1740 lb. per sq. in.; 1–3–6 concrete, 1030 lb. per sq. in.; 1–4–8 concrete, 575 lb. per sq. in. The effect of the amount of cement used in the concrete is very marked, and cement is shown to be an economical reinforcing material for compression members. The average variation of individual results from the average strength in the case of 1–2–4 concrete about 60 days old is 12%, ranging from 34% below to 25% above the average. The tests made at an age of 6 months indicate an

increase of something like 15% over the strength at 60 days.

2. The average initial modulus of elasticity of 1–2–4 concrete columns at an age of about 60 days was 3 040 000 lb. per sq. in., as compared with 2 250 000 lb. per sq. in. given for the columns in Bulletin No. 10. The larger value is due in part to the denser and harder concrete used.

3. Poisson's ratio for concrete in compression is a variable quantity, increasing considerably just before the ultimate failure of the concrete. The value varies with the age and nature of the concrete. A value of 0.1 to 0.16 for 1–2–4 concrete 60 days old may be tentatively given for the lower loads, with values as high as 0.25 or 0.3 near the crushing load. This concrete set in air.

4. The repetition of a load gives an increased amount of shortening and an increased set, the amount of this increase varying with the amount of the load. For the lower loads the increase becomes small after a small number of repetitions. For the higher loads the effect of continued repetition is much more marked. The phenomena attending repetitive loading of columns are important and merit much fuller investigation.

5. In hooped columns the hooping does not come into action to any great extent before a load equivalent to the ultimate strength of plain concrete, or a little below, is reached. The longitudinal deformation and the lateral deformation are not modified by the hooping to any great extent before this load is reached.

6. Beyond this load the column shortens rapidly and the hooping receives additional stress correspondingly and its function in restraining the concrete laterally becomes important. If the column maintains a straight position, the limit of strength may be expected to come when the stress in the hooping exceeds the yield point of the metal. If the column deflects laterally through eccentric loading or lack of uniformity in the concrete or in the distribution of the hooping, a lower strength will be found.

7. The additional strength of the hooped column over that for an unreinforced concrete column of the same quality, as determined from the stress-deformation diagram, averages for each one per cent of hooping 955 lb. per sq. in. for the spiral-hooped columns and 669 lb. per sq. in. for the band-hooped columns having a diameter of 12 in. The additional strength per one per cent of reinforcement is much the same for 1–4–8 concrete, 1–2–4 con-

crete, and 1-1½-3 concrete, but it must not be overlooked that the 1–4–8 concrete is much less stiff than are the columns with richer mixtures, especially after the hooping is brought into action. The difference between the strength of columns with high-carbon wire and with mild-steel wire is not marked enough to warrant drawing conclusions.

8. For loads which bring the hooping into full action, the increase in the longitudinal deformation and in the lateral deformation are approximately proportional to the added load. The ratio of lateral to longitudinal deformation is more nearly constant than for plain concrete. Through this stage the ratio of increment of load to increment of longitudinal shortening is low, the averages for columns with band-hoops being 184 000 and with spiral-hoops 165 000. This ratio is very much less than the modulus of elasticity of concrete and the hooped columns through this stage of loading are correspondingly less stiff than concrete columns. The amount of the set produced by the higher loads is very great.

9. The total amount of shortening before failure occurs is very great, averaging something like six to twelve times that for plain concrete and fifty times that for the ordinary working stresses in concrete. At the maximum load it is somewhat less than this, say five times as much as plain concrete and perhaps five times that of mild steel at its elastic limit. Cracking and peeling of the concrete are apparent at loads corresponding to the ultimate strength of concrete.

10. The excessive amount of compression before failure affects the problem of combining hooping and longitudinal reinforcement very unfavorably, if the stresses are to be kept within the elastic limit of the latter.

11. The lateral deflection of hooped columns is large and may seriously affect the ultimate strength which is available for the column. For continued application of stress beyond the maximum load, the column deflects enormously. Scaling of the surface of the concrete and lateral deflections are warning signs given well before danger of failure exists.

12. The concrete itself retains a considerable element of its strength even after it has been shortened in a hooped state four or five times as much as would produce failure in unhooped concrete columns.

13. Columns of rich and lean concrete exhibit phenomena of similar characteristics, the hoop stress becoming effective at the ultimate strength of unhooped concrete.

14. The one experiment indicates that hooped columns will resist eccentric stresses in somewhat the same way as will other material.

15. Light hooping offers security against sudden failure and unevenness of concrete and will enable higher working stresses to be used. In combination with rich concretes and longitudinal reinforcement, using low stresses in hoops (i. e., basing the strength upon an assumed ultimate strength but little beyond the average ultimate strength of plain concrete), a satisfactory column may be made. It is suggested that a column of this character may be designed in such a way that the longitudinal reinforcement may carry the load during construction and still not be overstressed later with the final loading, provided the basal point for use with the factor of safety in determining the working strength of the column is somewhat below the ultimate strength for plain concrete.

16. Heavy hooping gives added strength, but in utilizing the full strength of such columns the column shortens unduly, deflects laterally, and will strain longitudinal reinforcement many times beyond the deformation which exists at the elastic limit of the metal. It may be applicable where a large limit of safety is desired or where large variations in shortening are unobjectionable, as where the structure is articulated. So far as ultimate strength is concerned, hooping adds two to three times as much strength to the column as does an equal amount of longitudinal reinforcement, but with the extreme amount of shortening and the liability to lateral deflection it may be doubted whether this increase of strength may be utilized to any great extent in ordinary construction.

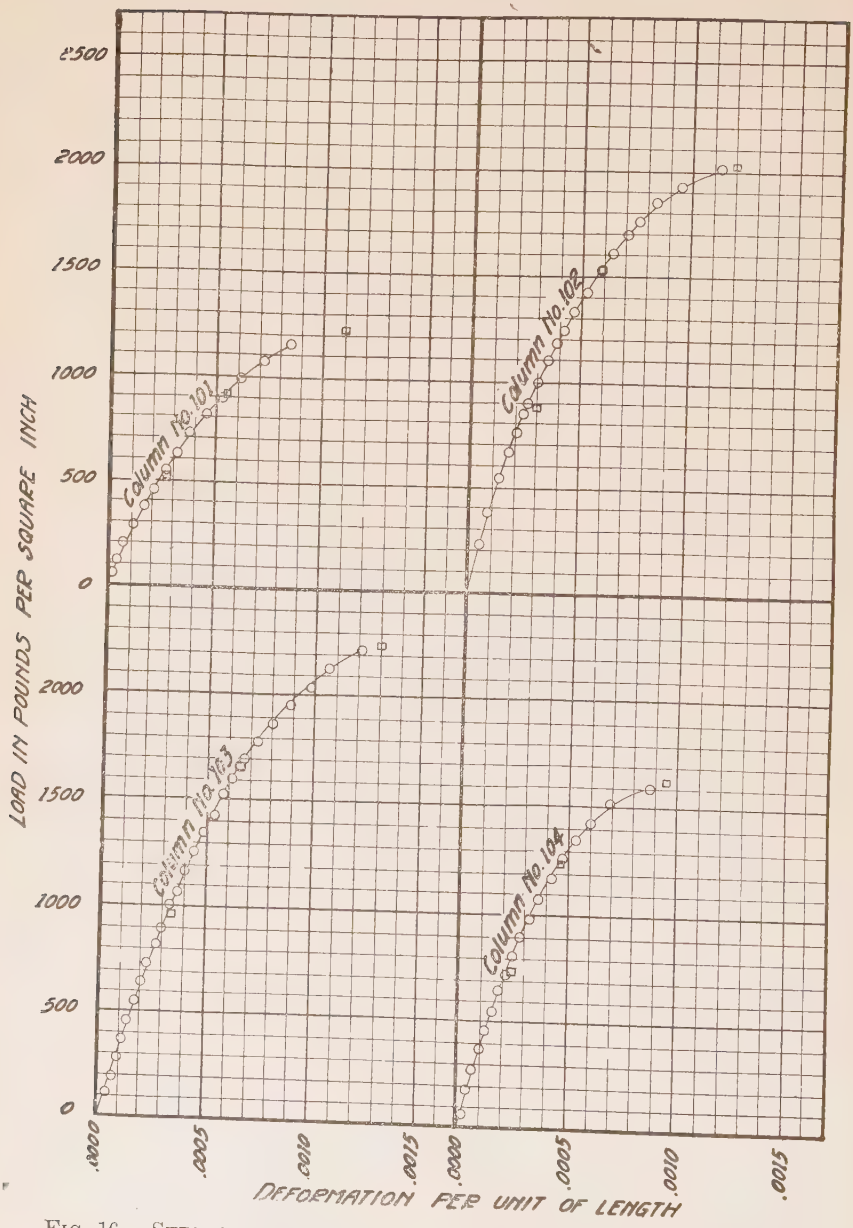

FIG. 16. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

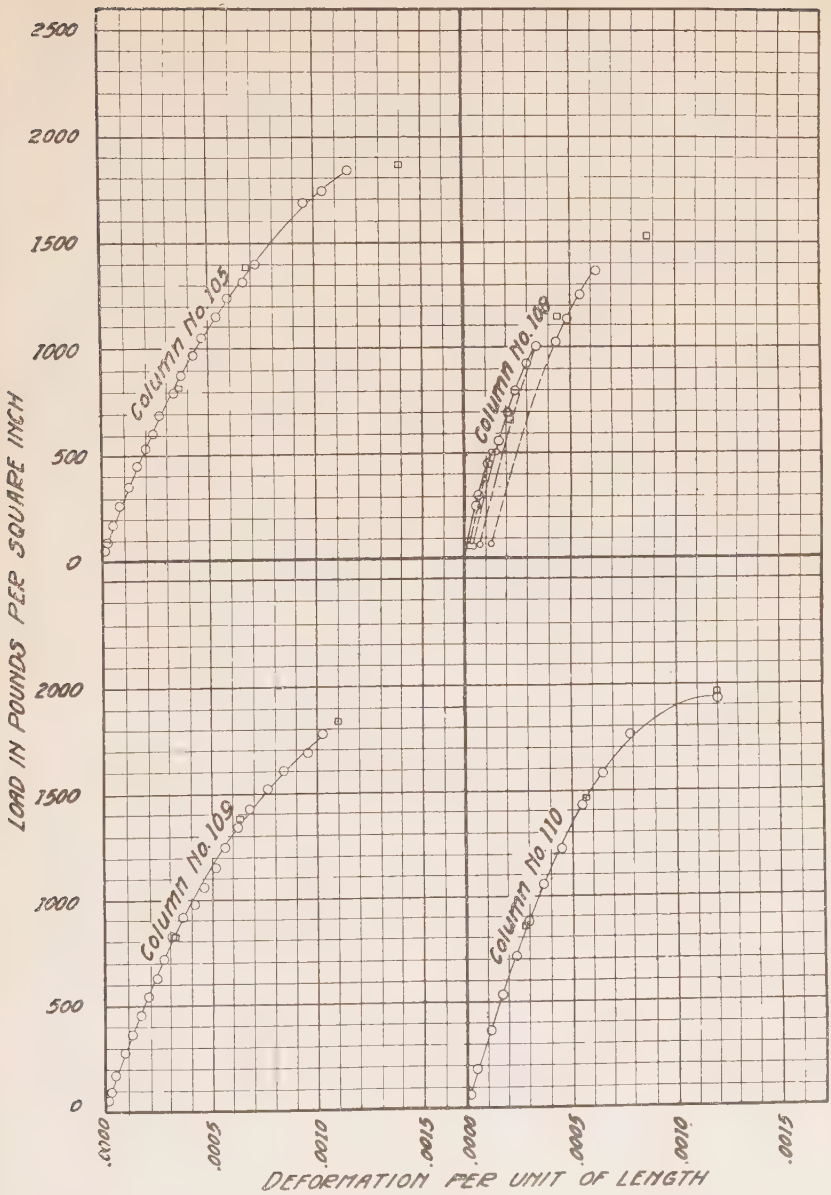

FIG. 17. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

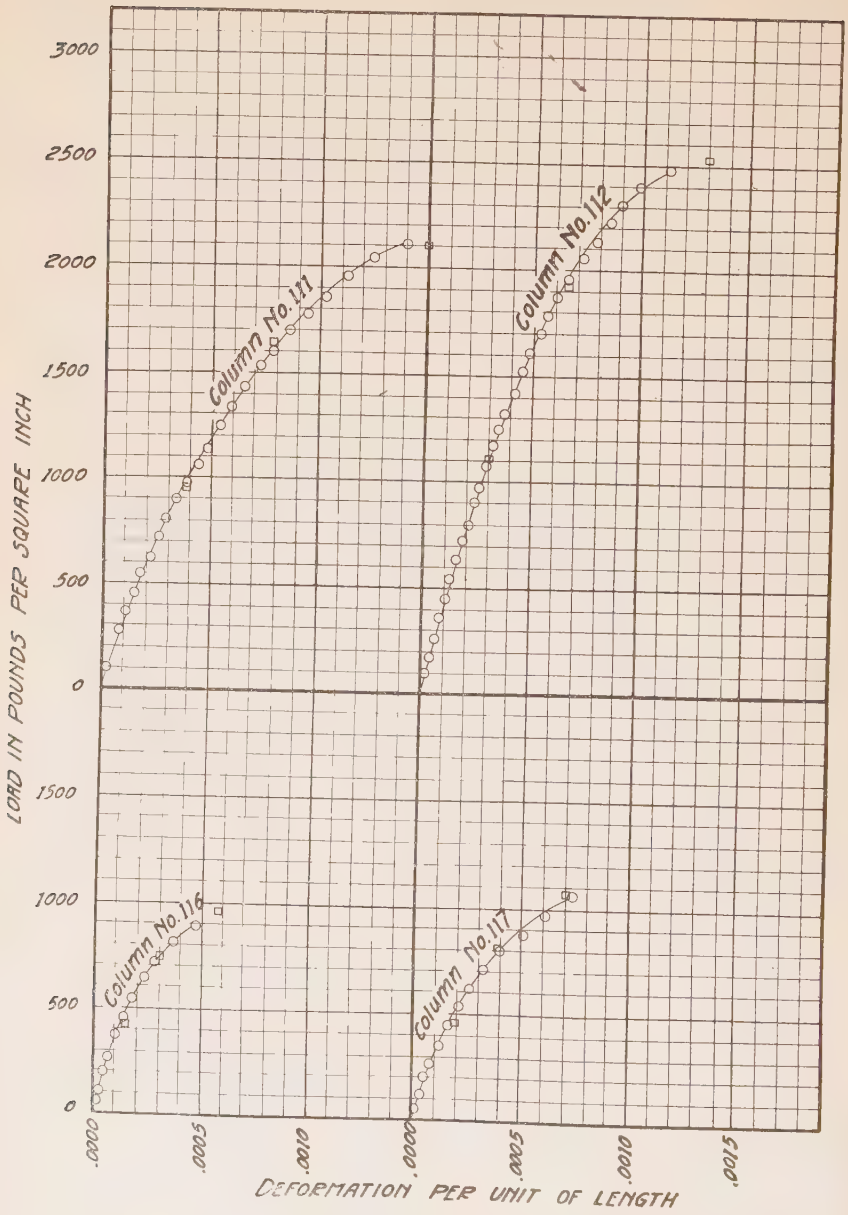

FIG. 18. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

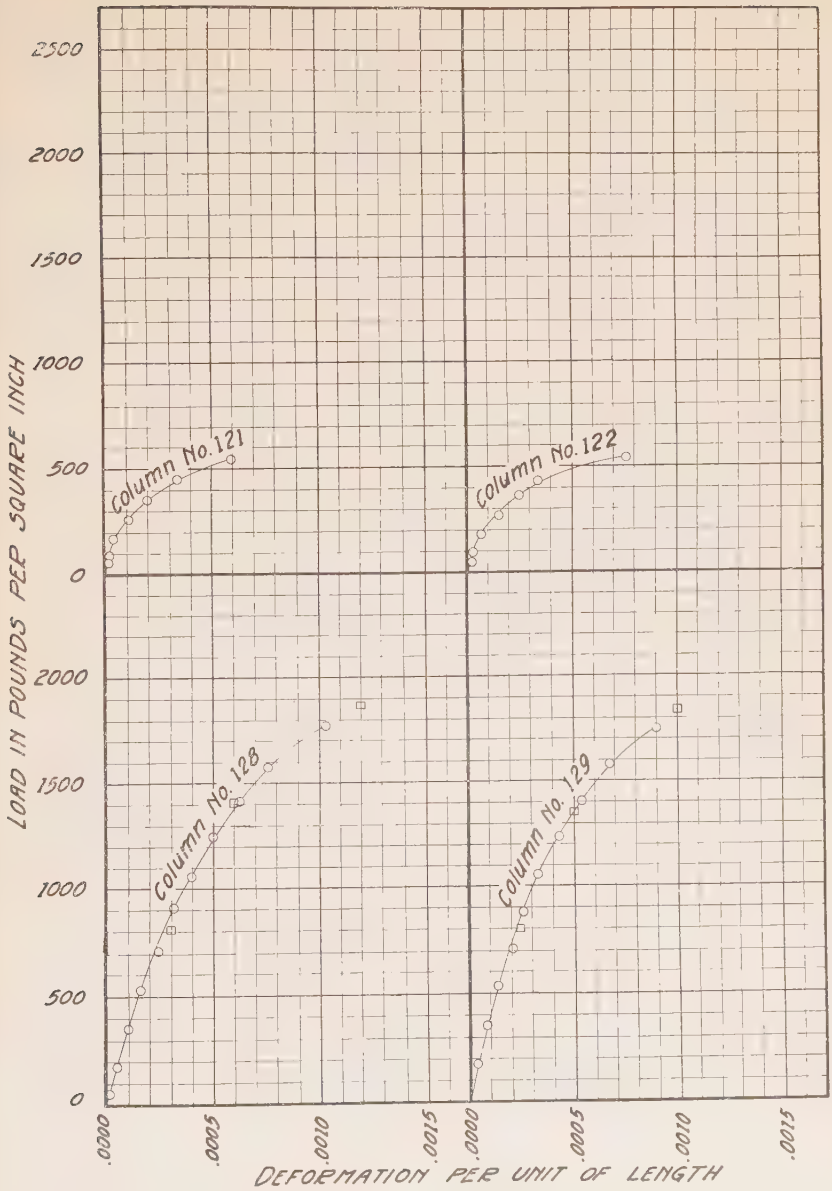

FIG. 19. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

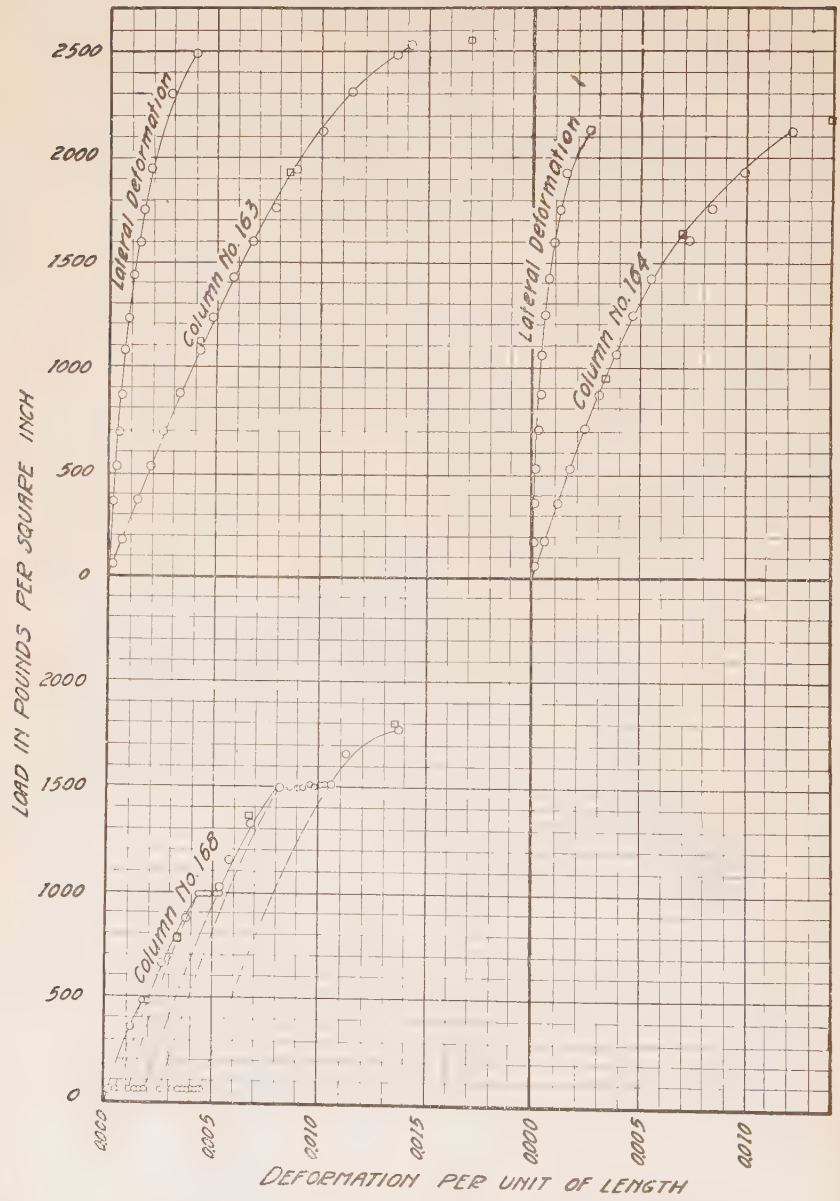

FIG. 20. STRESS-DEFORMATION DIAGRAMS FOR CONCRETE COLUMNS.

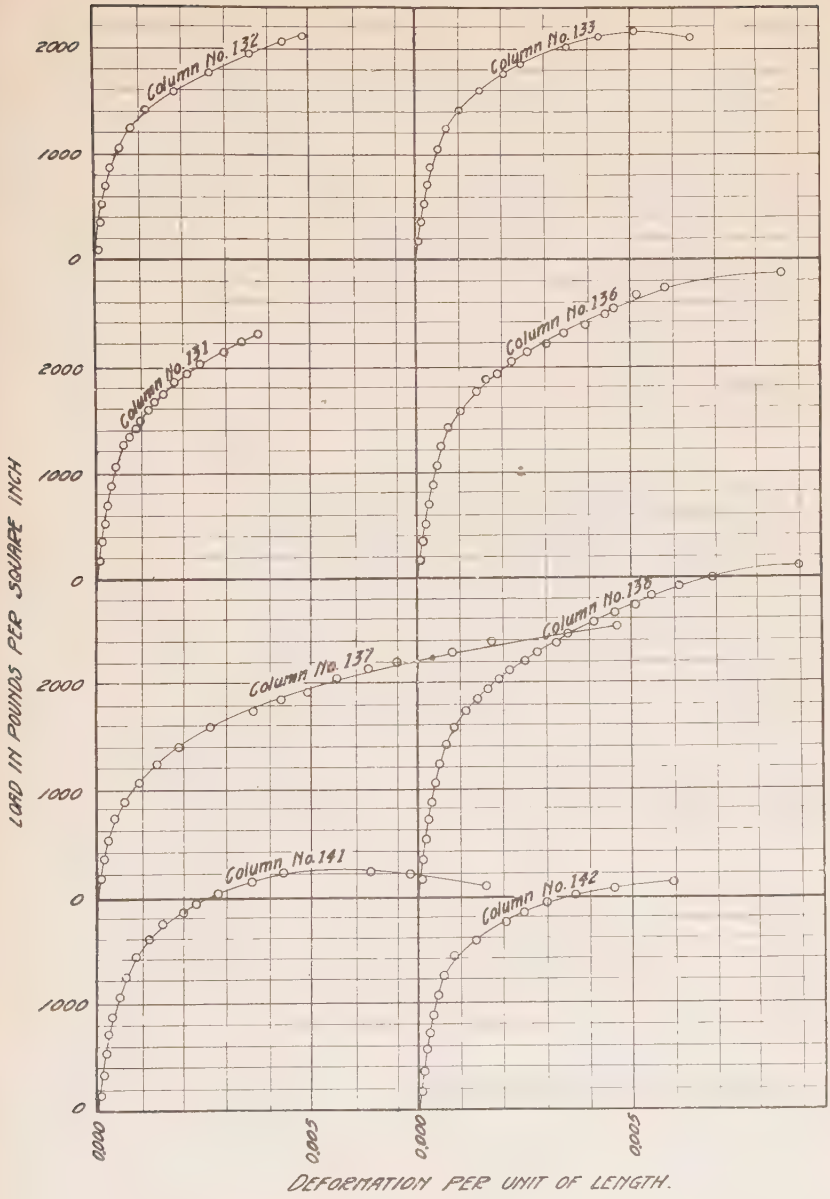

FIG. 21. STRESS-DEFORMATION DIAGRAMS FOR BAND-HOOPED COLUMNS.

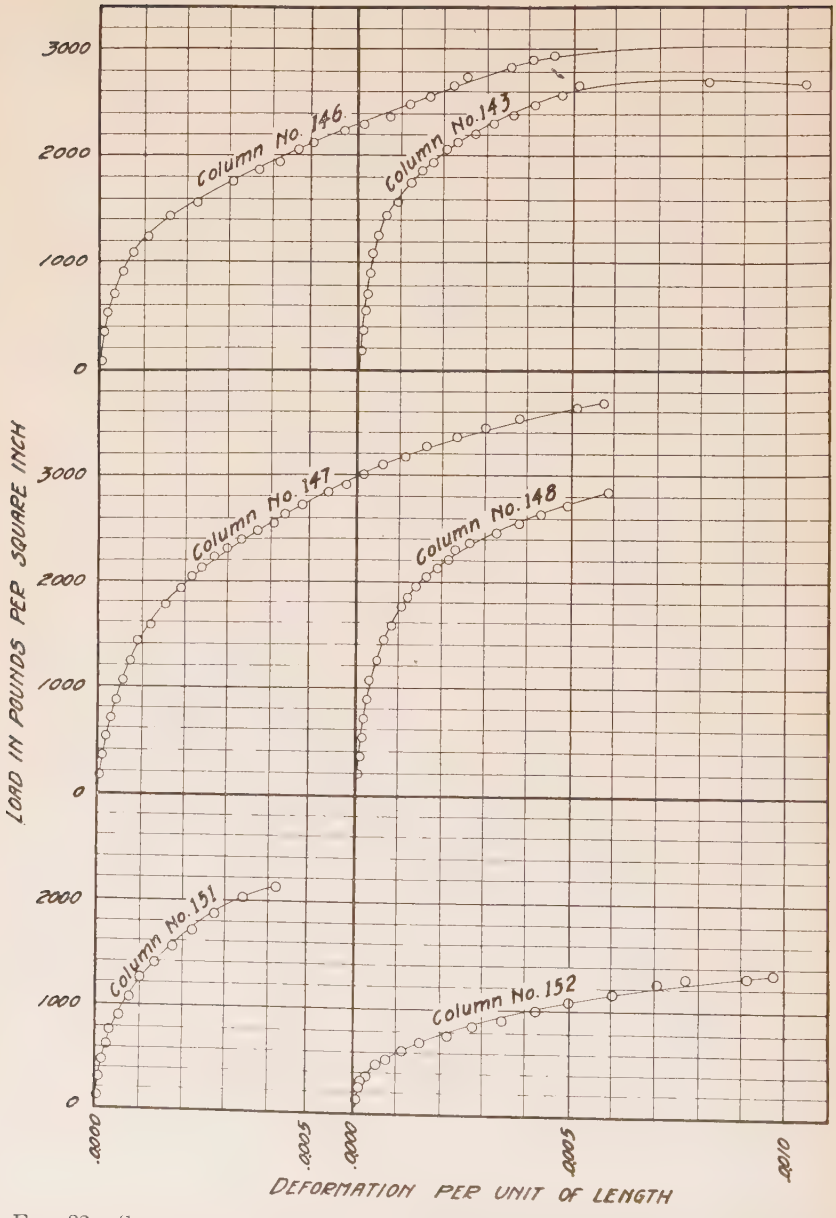

FIG. 22. STRESS-DEFORMATION DIAGRAMS FOR BAND-HOOPED COLUMNS.

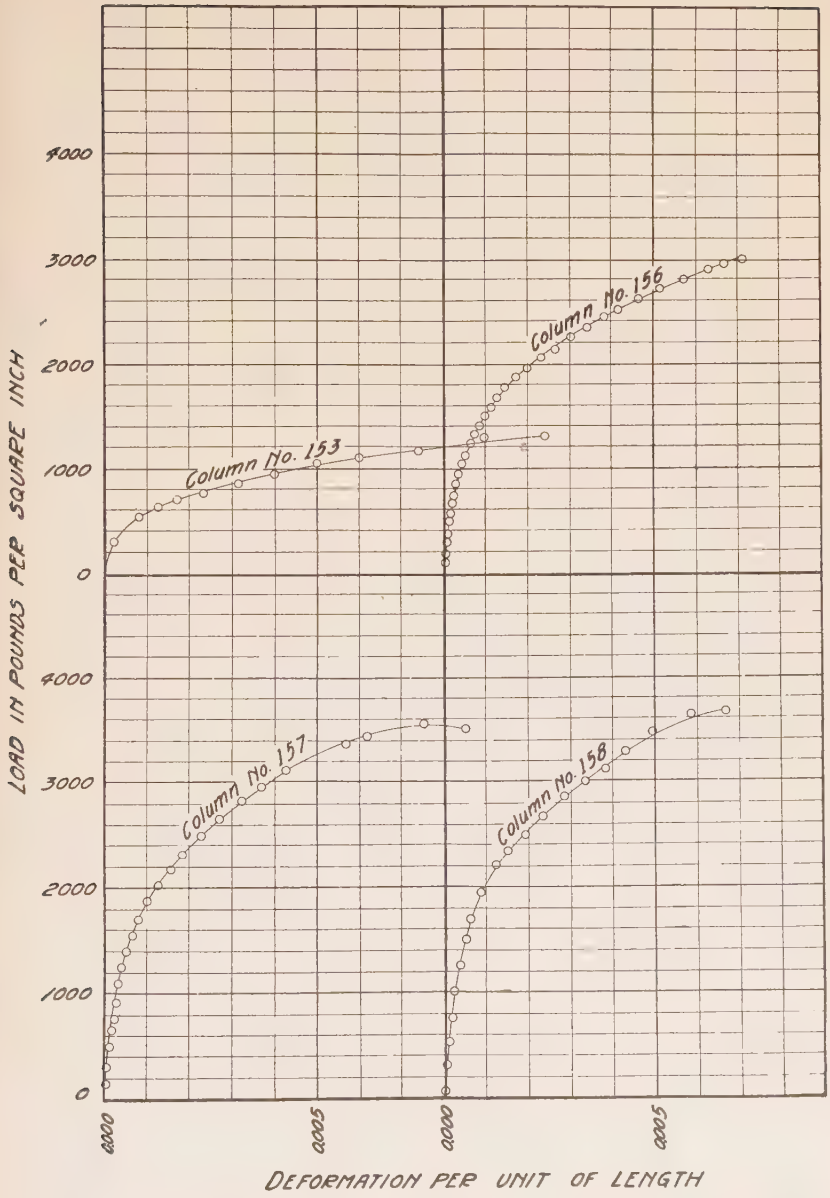

FIG. 23. STRESS-DEFORMATION DIAGRAMS FOR BAND-HOOPED COLUMNS.

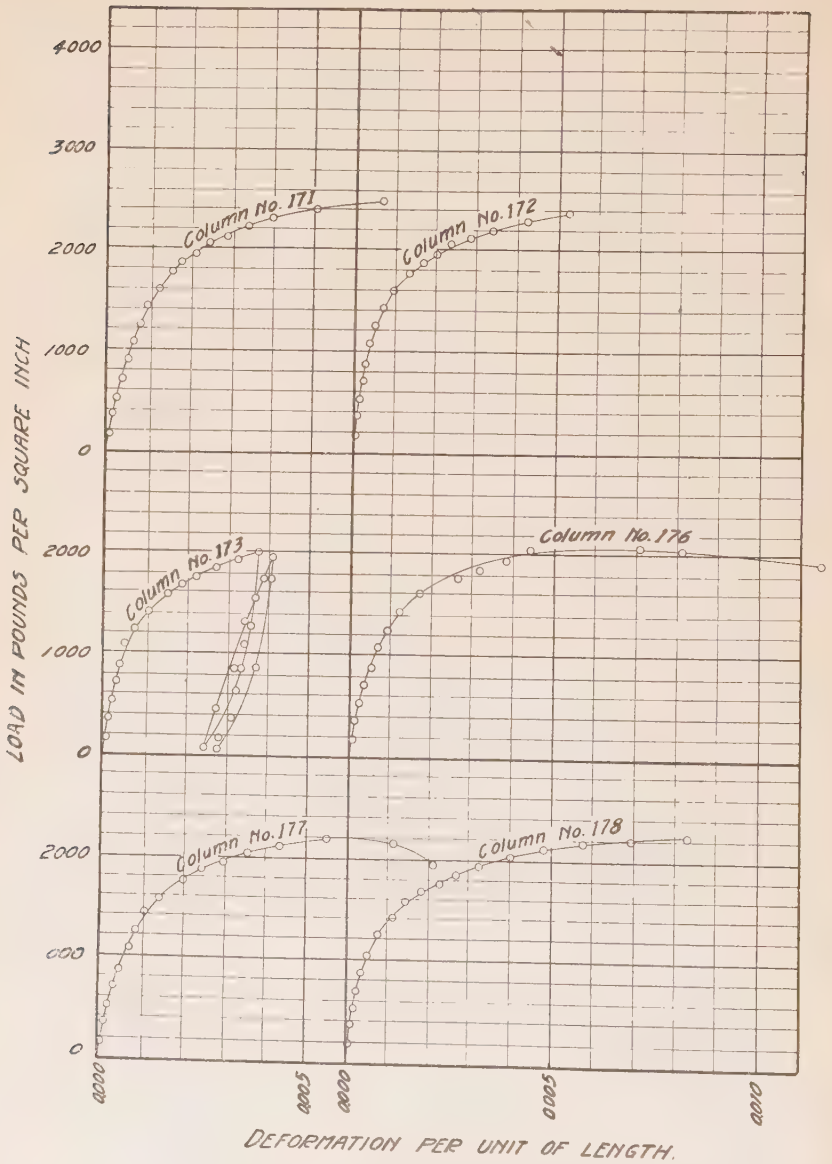

FIG. 24. STRESS-DEFORMATION DIAGRAMS FOR SPIRAL-HOOPED COLUMNS.

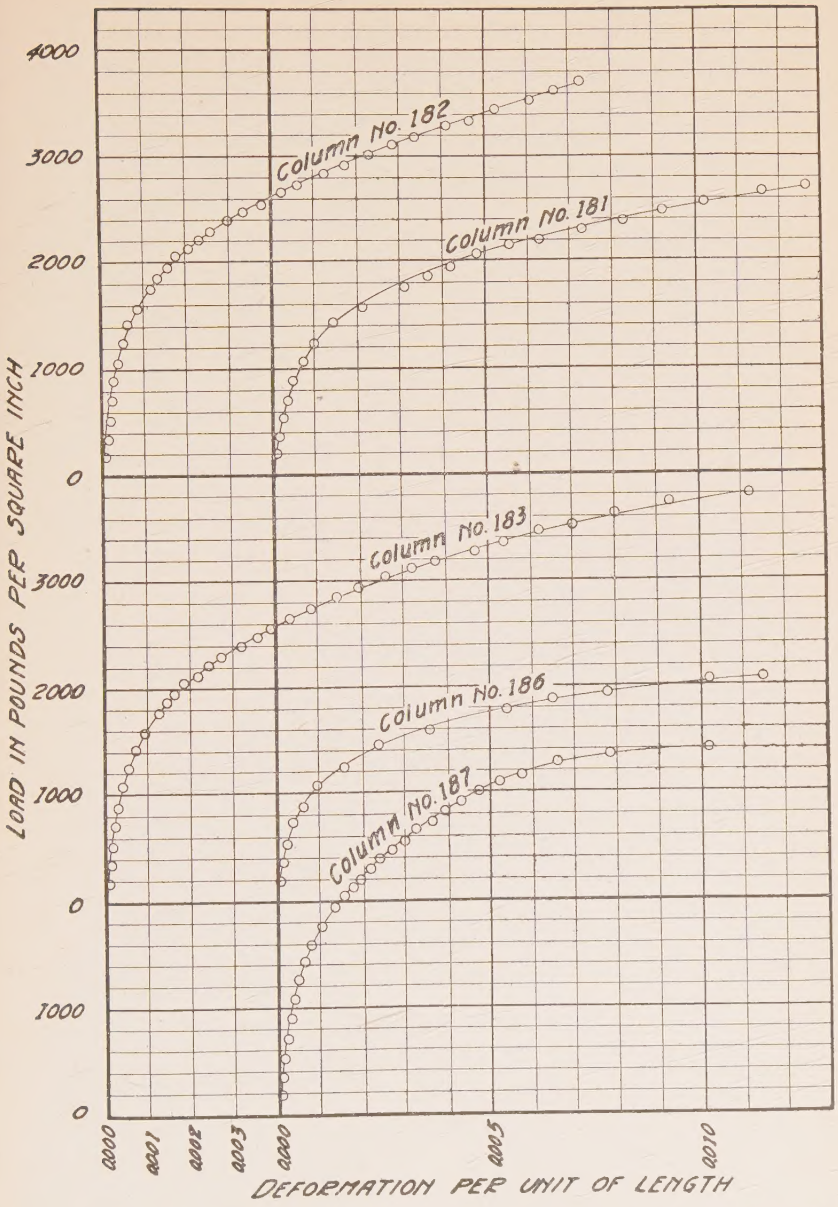

FIG. 25. STRESS-DEFORMATION DIAGRAMS FOR SPIRAL-HOOPED COLUMNS.

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